

Recent developments in parton showers

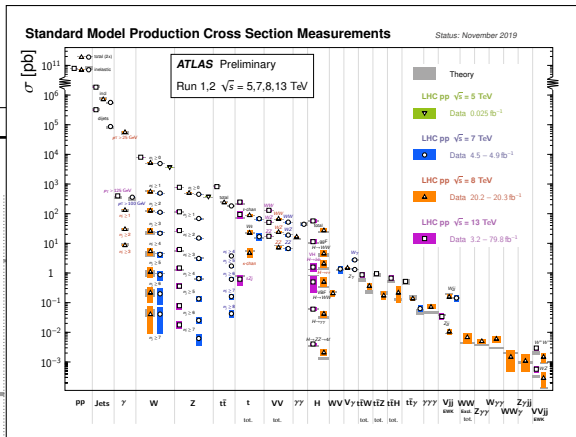
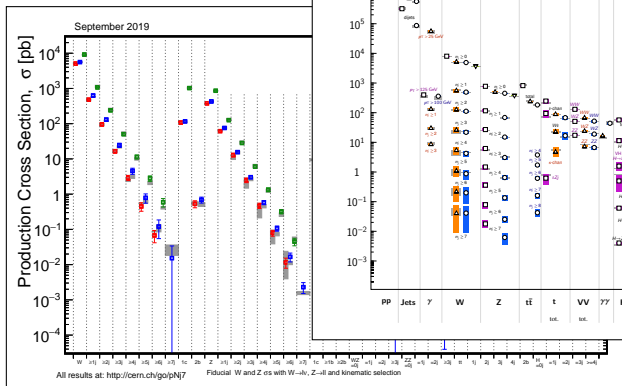
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Zürich, 13/1/2023

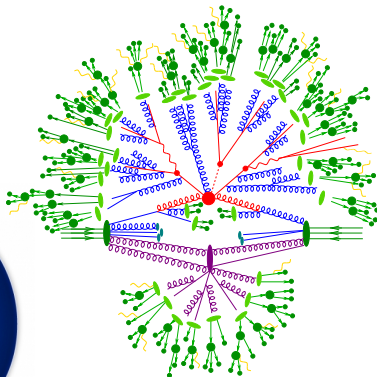
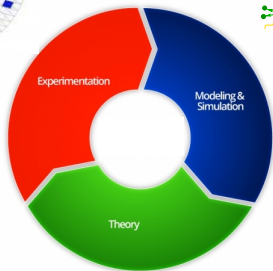
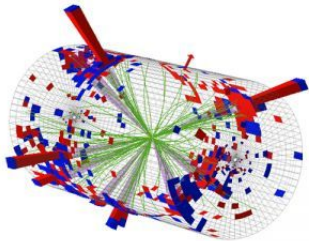
The Standard Model as we know it



[ATLAS] <https://twiki.cern.ch/twiki/bin/view/AtlasPublic/StandardModelPublicResults>

[CMS] <https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsCombined>

How we measure it



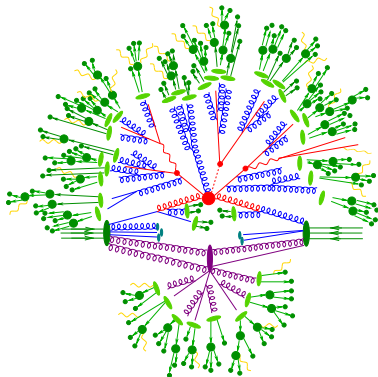
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi}\not{D}\psi + h.c.$$

LHC event generators

[Buckley et al.] arXiv:1101.2599

[Campbell et al.] arXiv:2203.11110

- Short distance interactions
 - Signal process
 - Radiative corrections
- Long-distance interactions
 - Hadronization
 - Particle decays



Divide and Conquer

- Quantity of interest: Total interaction rate
- Convolution of short & long distance physics

$$\sigma_{p_1 p_2 \rightarrow X} = \sum_{i,j \in \{q,g\}} \int dx_1 dx_2 \underbrace{f_{p_1,i}(x_1, \mu_F^2) f_{p_2,j}(x_2, \mu_F^2)}_{\text{long distance}} \underbrace{\hat{\sigma}_{ij \rightarrow X}(x_1 x_2, \mu_F^2)}_{\text{short distance}}$$

Connection to QCD theory

- $\hat{\sigma}_{ij \rightarrow n}(\mu_F^2)$ → Collinearly factorized fixed-order result at N^xLO

Implemented in fully differential form to be maximally useful

Tree level: $d\Phi_n B_n$

- Automated ME generators + phase-space integrators

1-Loop level: $d\Phi_n \left(B_n + V_n + \sum C + \sum I_n \right) + d\Phi_{n+1} \left(R_n - \sum S_n \right)$

- Automated loop ME generators + integral libraries + IR subtraction

2-Loop level: It depends ...

- Individual solutions based on SCET, q_T subtraction, P2B

- $f_i(x, \mu_F^2)$ → Collinearly factorized PDF at N^yLO

Evaluated at $O(1\text{GeV}^2)$ and expanded into a series above 1GeV^2

$$\text{DGLAP: } \frac{dx x f_a(x, t)}{d \ln t} = \sum_{b=q,g} \int_0^1 d\tau \int_0^1 dz \frac{\alpha_s}{2\pi} [z P_{ab}(z)]_+ \tau f_b(\tau, t) \delta(x - \tau z)$$

- Parton showers, dipole showers, antenna showers, ...

$$\text{Matching: } d\Phi_n \frac{S_n}{B_n} \leftrightarrow \frac{dt}{t} dz \frac{\alpha_s}{2\pi} P_{ab}(z)$$

- MC@NLO, POWHEG, Geneva, MINNLO_{PS}, ...

Directions of development

Much effort focused on parton-shower component recently

- Phenomenologically interesting: Drives jet production, b -tagging, ...
- Experimentally relevant: Often source of largest uncertainty
- The unique part of event generators (in perturbative QCD)

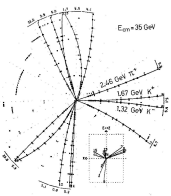
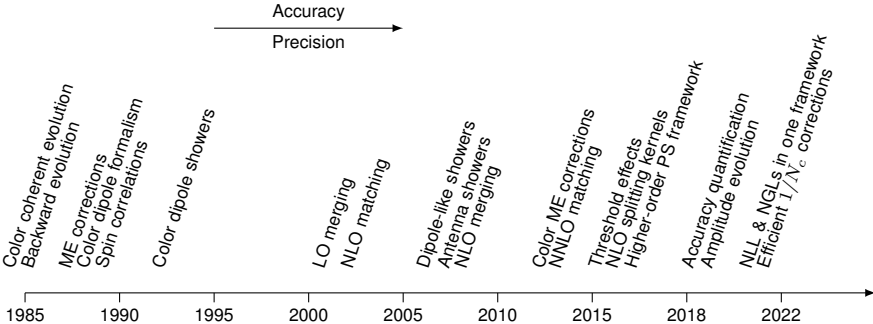
Fixed-order aspects

- Matching to NLO calculations
 - Negative weight fraction
 - Unweighting efficiency
- Matching to NNLO calculations
 - Semi-inclusive
 - Fully differential
- Matching to N³LO calculations

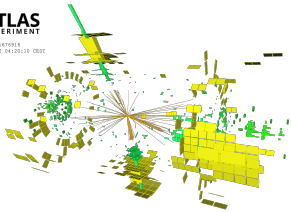
All-order aspects

- NLL precision
- Splitting functions at NLO
- Spin correlations
- Sub-leading power corrections
- Sub-leading color effects
- Threshold effects
- Amplitude evolution

Evolution of parton-showers over time



$\sqrt{s} \times 500$
 e^+e^- vs. pp



22.8.00

Simulation of QCD dipole radiation

Approaches, problems & solutions

Semi-classical radiation pattern

[Marchesini,Webber] NPB310(1988)461

- Soft gluon radiator can be written in terms of energies and angles

$$J_\mu J^\mu \rightarrow \frac{p_i p_k}{(p_i p_j)(p_j p_k)} = \frac{W_{ik,j}}{E_j^2}$$

Angular “radiator” function

$$W_{ik,j} = \frac{1 - \cos \theta_{ik}}{(1 - \cos \theta_{ij})(1 - \cos \theta_{jk})}$$

- Divergent as $\theta_{ij} \rightarrow 0$ and as $\theta_{jk} \rightarrow 0$

→ Expose individual collinear singularities using $W_{ik,j} = \tilde{W}_{ik,j}^i + \tilde{W}_{ik,j}^k$

$$\tilde{W}_{ik,j}^i = \frac{1}{2} \left[\frac{1 - \cos \theta_{ik}}{(1 - \cos \theta_{ij})(1 - \cos \theta_{kj})} + \frac{1}{1 - \cos \theta_{ij}} - \frac{1}{1 - \cos \theta_{kj}} \right]$$

- Divergent as $\theta_{ij} \rightarrow 0$, but regular as $\theta_{kj} \rightarrow 0$
- Convenient properties upon integration over azimuthal angle

Semi-classical radiation pattern

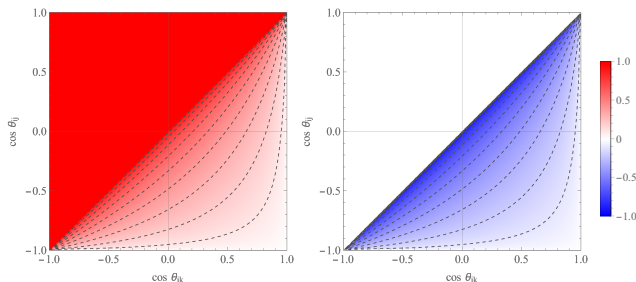
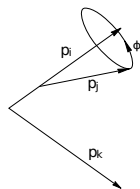
- Work in a frame where direction of \vec{p}_i aligned with z -axis

$$\cos \theta_{kj} = \cos \theta_k^i \cos \theta_j^i + \sin \theta_k^i \sin \theta_j^i \cos \phi_{kj}^i$$

- Integration over ϕ_j yields

$$\frac{1}{2\pi} \int_0^{2\pi} d\phi_{kj}^i \tilde{W}_{ik,j}^i = \frac{1}{1 - \cos \theta_j^i} \times \begin{cases} 1 & \text{if } \theta_j^i < \theta_k^i \\ 0 & \text{else} \end{cases}$$

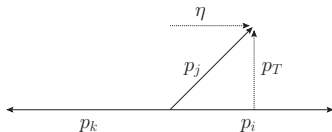
- On average, no radiation outside cone defined by parent dipole
- Differential radiation pattern more intricate:
Positive & negative contributions outside cone sum to zero



Dual description and the Lund plane

[Gustafson] PLB175(1986)453

- Compute everything in center-of-mass frame of fast partons



- Simple expressions for transverse momentum and rapidity

$$p_T^2 = \frac{2(p_i p_j)(p_k p_j)}{p_i p_k}, \quad \eta = \frac{1}{2} \ln \frac{p_i p_j}{p_k p_j}$$

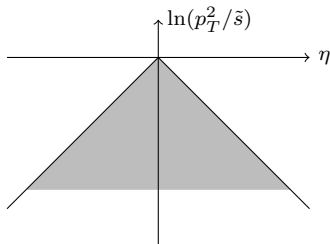
- In momentum conserving parton branching $(\tilde{p}_i, \tilde{p}_k) \rightarrow (p_i, p_k, p_j)$

$$-\ln \tilde{s}_{ik}/p_T^2 \leq 2\eta \leq \ln \tilde{s}_{ik}/p_T^2$$

- Differential phase-space element $\propto dp_T^2 d\eta$

- Visualized in Lund plane

- Phase space bounded by diagonals
- Single-emission semi-classical radiation probability a constant



Angular ordered parton showers

[Marchesini,Webber] NPB238(1984)1, ...

■ Differential radiation probability

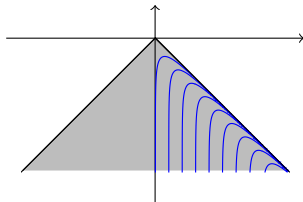
$$d\mathcal{P} = d\Phi_{+1} |M|^2 \approx \frac{d\tilde{q}^2}{\tilde{q}^2} dz \frac{\alpha_s}{2\pi} P_{i\tilde{j}i}(z)$$

- Ordering parameter $\tilde{q}^2 = \frac{2p_i p_j}{z(1-z)} \approx 4E_{ij}^2 \sin^2 \frac{\theta_{ij}}{2}$

- Splitting variable $z = \frac{1 + \cos \theta_{ik}}{2} = \frac{p_i p_k}{(p_i + p_j) p_k}$

■ Lund plane filled from center to edges

- Random walk in p_T^2
- Color factors correct for observables insensitive to azimuthal correlations
- Small dead zone at $\ln(p_T^2/\bar{s}) \approx 0$



- Usually disfavored due to dead zones
Not suitable to resum non-global logarithms

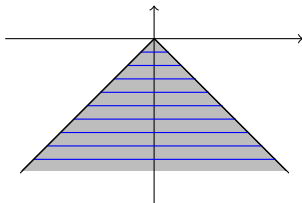
Dipole showers

[Gustafson, Pettersson] NPB306(1988)746, ...

- Differential radiation probability for the dipole

$$d\mathcal{P} = d\Phi_{+1} |M|^2 \approx \frac{dp_T^2}{p_T^2} d\eta \frac{\alpha_s}{2\pi} \tilde{P}_{\tilde{\gamma}}(z)$$

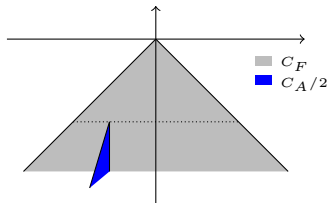
- Ordering parameter p_T^2
- Splitting variable $z = 1 - \frac{s_{ij}}{s - s_{ij}} e^{-2\eta}$
- Lund plane filled from top to bottom
 - Random walk in η
 - Color factors in CFFE approximation
 - Pairs of partons evolve simultaneously
 - No dead zones
- Solves problem of dead zones
Known issues with color coherence



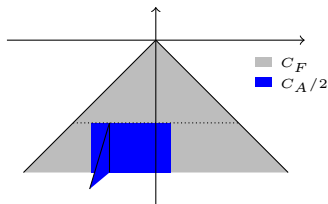
Problems with average color charges

[Gustafsson] NPB392(1993)251

- In angular ordered showers angles are measured in the event center-of-mass frame
→ coherence effects modeled by angular ordering variable agree on average with matrix element



- In dipole-like showers angles effectively measured in center-of-mass frame of emitting color dipole
→ angular coherence not reflected by setting average QCD charge



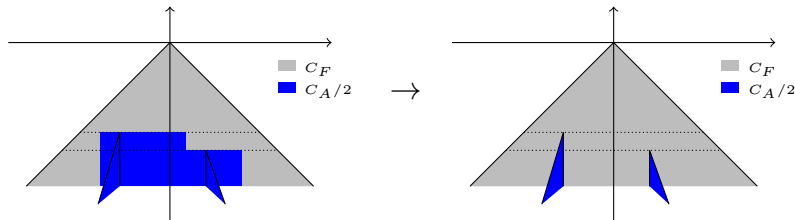
- Emission off “back plane” in Lund diagram should be associated with C_F , but is partly associated with $C_A/2$ in dipole showers
- All-orders problem that appears first in 2-gluon emission case

Solutions for average color charges

[Gustafsson] NPB392(1993)251

[Hamilton,Medves,Salam,Scyboz,Soyez] arXiv:2011.10054

- Analyze rapidity of gluon emission in event center-of-mass frame
- Sectorize phase space and assign gluon to closest parton
→ choose corresponding color charge for evolution
- Same technology for higher number of emissions



- Starting with 4 emissions, there be “color monsters”

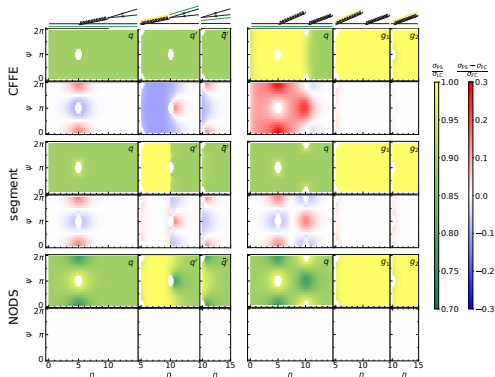
[Dokshitzer,Troian,Khoze] SJNP47(1988)881, YF47(1988)1384

- Quartic Casimir operators (easy)
- Non-factorizable contributions (hard)

Solutions for average color charges

[Hamilton,Medves,Salam,Scyboz,Soyez] arXiv:2011.10054

- Can include double-soft corrections via reweighting [Giele,Kosower,Skands] arXiv:1102.2126
- Algorithm scales as N^2 but can be simplified while retaining formal accuracy
- Implementation as nested corrections in rapidity segments of parent dipole
- Excellent agreement with full matrix element
- Good agreement with full-color evolution [Hatta,Ueda] arXiv:1304.6930



Problems with momentum mapping

[Dasgupta,Dreyer,Hamilton,Monni,Salam] arXiv:1805.09327

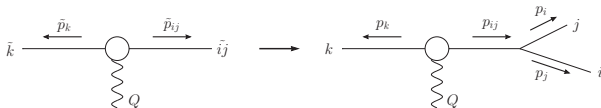
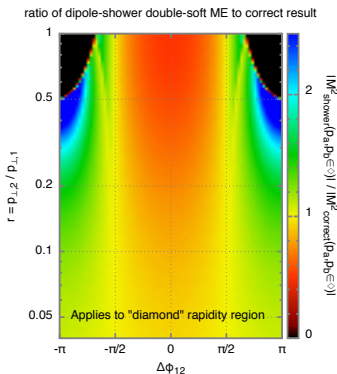
- Subtle problems in standard dipole-like momentum mapping

$$p_k^\mu = \left(1 - \frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\right) \tilde{p}_k^\mu$$

$$p_i^\mu = \tilde{z}\tilde{p}_{ij}^\mu + (1 - \tilde{z})\frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\tilde{p}_k^\mu + k_\perp^\mu$$

$$p_j^\mu = (1 - \tilde{z})\tilde{p}_{ij}^\mu + \tilde{z}\frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\tilde{p}_k^\mu - k_\perp^\mu$$

- Induces angular correlations across multiple emissions
- Spoils agreement w/ analytic resummation



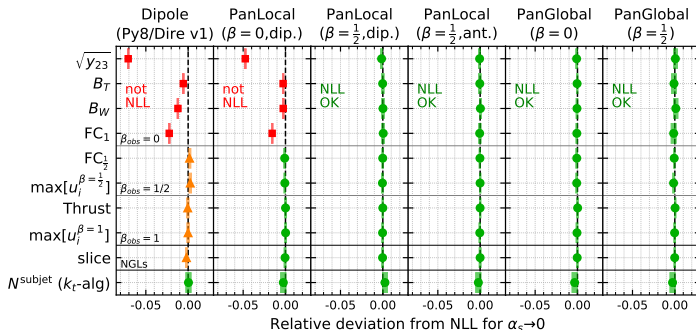
Solutions for momentum mapping

[Dasgupta,Dreyer,Hamilton,Monni,Salam,Soyez] arXiv:2002.11114

- Problem can be solved by partitioning of antenna radiation pattern and choosing a suitable evolution variable ($\beta \sim 1/2$)

$$k_T = \rho v e^{\beta|\bar{\eta}|} \quad \rho = \left(\frac{s_i s_j}{Q^2 s_{ij}} \right)^{\beta/2}$$

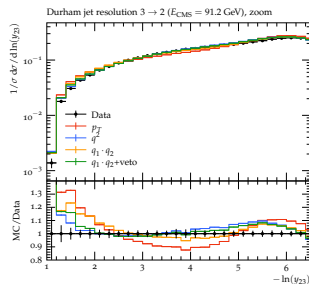
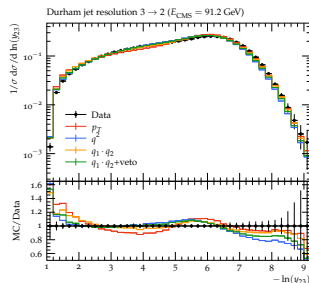
- Different recoil schemes can lead to NLL result if β chosen appropriately: Local dipole, local antenna, and global antenna
- NLL correct for global and non-global observables in $e^+e^- \rightarrow \text{hadrons}$



Solutions for momentum mapping

[Bewick,Ferrario-Ravasio,Richardson,Seymour] arXiv:1904.11866

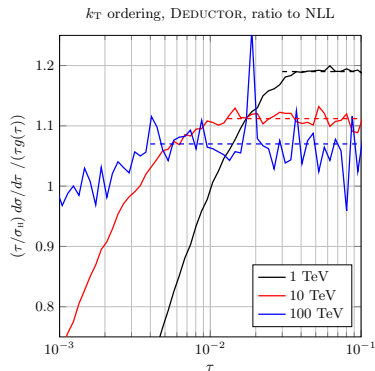
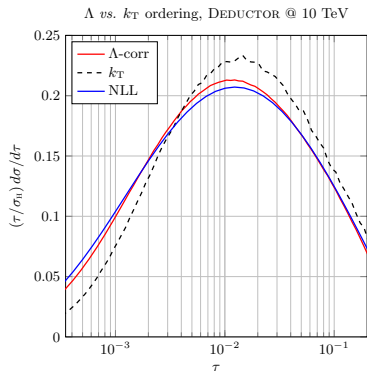
- Recoil schemes affect logarithmic accuracy but impact also phase-space coverage
- In context of angular ordered Herwig 7 (NLL accurate for global observables)
 - q_T preserving scheme:
 - Maintains logarithmic accuracy
 - Overpopulates hard region
 - q^2 preserving scheme:
 - Breaks logarithmic accuracy
 - Good description of hard region
 - Dot product preserving scheme (new):
 - Maintains logarithmic accuracy
 - Good description of hard radiation



Solutions for momentum mapping

[Nagy,Soper] arXiv:2011.04773

- Local transverse recoil, global longitudinal recoil
- Analytic proof of NLL correctness, based on kinematics in $s \rightarrow \infty$ limit



A new perspective on old ideas

Identified partons & azimuthal angle dependence

The semi-classical matrix element revisited

- Alternative to additive matching: partial fraction matrix element & match to collinear sectors [Ellis,Ross,Terrano] NPB178(1981)421, [Catani,Seymour] hep-ph/9605323

$$\frac{W_{ik,j}}{E_j^2} \rightarrow \frac{1}{p_i p_j} \frac{p_i p_k}{(p_i + p_k) p_j} + \frac{1}{p_k p_j} \frac{p_i p_k}{(p_i + p_k) p_j}$$

- Captures matrix element both in angular ordered and unordered region
 - Caveat: Oversampling difficult for certain kinematics maps
- Separate into energy & angle first [Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057
 Partial fraction angular radiator only: $W_{ik,j} = \bar{W}_{ik,j}^i + \bar{W}_{ki,j}^k$

$$\bar{W}_{ik,j}^i = \frac{1 - \cos \theta_{ik}}{(1 - \cos \theta_{ij})(2 - \cos \theta_{ij} - \cos \theta_{kj})}$$

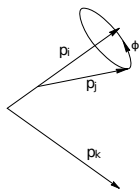
- Bounded by $(1 - \cos \theta_{ij}) \bar{W}_{ik,j}^i < 2$
- Strictly positive

The semi-classical matrix element revisited

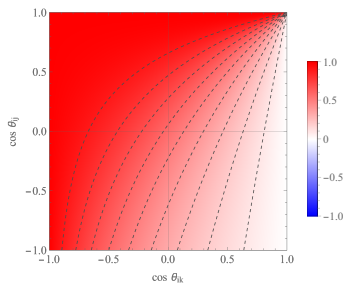
- Integration over ϕ_j yields

$$\frac{1}{2\pi} \int_0^{2\pi} d\phi_{kj}^i \bar{W}_{ik,j}^i = \frac{1}{1 - \cos \theta_j^i} \frac{1}{\sqrt{(\bar{A}_{ij,k}^i)^2 - (\bar{B}_{ij,k}^i)^2}}$$

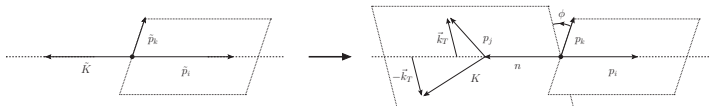
- Radiation across all of phase space
- Probabilistic radiation pattern



$$\bar{A}_{ij,k}^i = \frac{2 - \cos \theta_j^i (1 + \cos \theta_k^i)}{1 - \cos \theta_k^i}$$
$$\bar{B}_{ij,k}^i = \frac{\sqrt{(1 - \cos^2 \theta_j^i)(1 - \cos^2 \theta_k^i)}}{1 - \cos \theta_k^i}$$



Kinematics mapping revisited



- In collinear limit, splitting kinematics defined by ($n \rightarrow$ auxiliary vector)

$$p_i \xrightarrow{i||j} z \tilde{p}_i, \quad p_j \xrightarrow{i||j} (1-z) \tilde{p}_i \quad \text{where} \quad z = \frac{p_i n}{(p_i + p_j) n}$$

- Parametrization, using hard momentum \tilde{K}

$$p_i = z \tilde{p}_i, \quad n = \tilde{K} + (1-z) \tilde{p}_i$$

- Using on-shell conditions & momentum conservation ($\kappa = \tilde{K}^2 / (2\tilde{p}_i \tilde{K})$)

$$p_j = (1-z) \tilde{p}_i + v(\tilde{K} - (1-z + 2\kappa) \tilde{p}_i) + k_{\perp}$$

$$K = \tilde{K} - v(\tilde{K} - (1-z + 2\kappa) \tilde{p}_i) - k_{\perp}$$

- Momenta in \tilde{K} Lorentz-boosted to new frame K [Catani,Seymour] hep-ph/9605323

$$p_l^{\mu} \rightarrow \Lambda_{\nu}^{\mu}(K, \tilde{K}) p_l^{\nu}, \quad \Lambda_{\nu}^{\mu}(K, \tilde{K}) = g_{\nu}^{\mu} - \frac{2(K + \tilde{K})^{\mu}(K + \tilde{K})_{\nu}}{(K + \tilde{K})^2} + \frac{2\tilde{K}^{\mu} K_{\nu}}{K^2}.$$

Logarithmic accuracy – Analytic proof

- Logarithmic accuracy of parton shower can be quantified by comparing results to (semi-)analytic resummation e.g. [Banfi,Salam,Zanderighi] hep-ph/0407286
- Example: Thrust or FC_0 in $e^+e^- \rightarrow \text{hadrons}$
- Define a shower evolution variable $\xi = k_T^2/(1-z)$
- Parton-shower one-emission probability for $\xi > Q^2\tau$

$$R_{\text{PS}}(\tau) = 2 \int_{Q^2\tau}^{Q^2} \frac{d\xi}{\xi} \int_{z_{\min}}^{z_{\max}} dz \frac{\alpha_s(k_T^2)}{2\pi} C_F \left[\frac{2}{1-z} - (1+z) \right] \Theta(\eta)$$

- Approximate to NLL accuracy

$$R_{\text{NLL}}(\tau) = 2 \int_{Q^2\tau}^{Q^2} \frac{d\xi}{\xi} \left[\int_0^1 dz \frac{\alpha_s(k_T^2)}{2\pi} \frac{2C_F}{1-z} \Theta(\eta) - \frac{\alpha_s(\xi)}{\pi} C_F B_q \right]$$

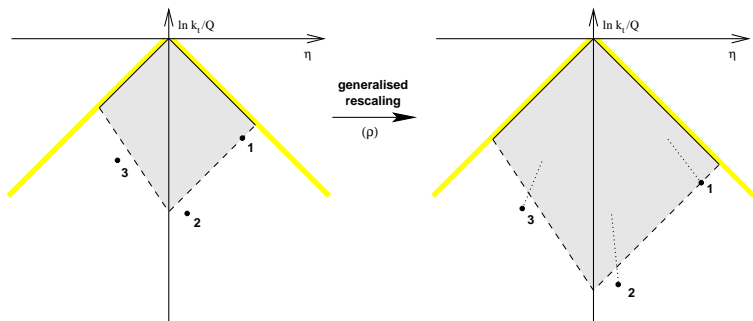
Logarithmic accuracy – Analytic proof

- Cumulative cross section $\Sigma(\tau) = e^{-R(\tau)} \mathcal{F}(\tau)$ obtained from all-orders resummed result by Taylor expansion of virtual corrections in cutoff ε

$$\mathcal{F}(\tau) = \int d^3 k_1 |M(k_1)|^2 e^{-R' \ln \frac{\tau}{\varepsilon v_1}} \sum_{m=0}^{\infty} \frac{1}{m!} \left(\prod_{i=2}^{m+1} \int_{\varepsilon v_1}^{v_1} d^3 k_i |M(k_i)|^2 \right) \times \Theta(\tau - V(\{p\}, k_1, \dots, k_n))$$

- $\mathcal{F}(\tau)$ is pure NLL & accounts for (correlated) multiple-emission effects
- In order to make $\mathcal{F}(\tau)$ calculable, make the following assumptions
 - Observable is recursively infrared and collinear safe
 - Hold $\alpha_s(Q^2) \ln \tau$ fixed, while taking limit $\tau \rightarrow 0$
 - Can factorize integrals and neglect kinematic edge effects
- Can be interpreted as $\alpha_s \rightarrow 0$ or $s \rightarrow \infty$ limit**

Logarithmic accuracy – Analytic proof



- $\alpha_s \rightarrow 0 / s \rightarrow \infty$ limit taken by similarity transformation of Lund plane
- Can be parametrized in terms of scaling parameter ρ

$$k_{t,l} \rightarrow k'_{t,l} = k_{t,l} \rho^{(1-\xi_l)/a + \xi_l/(a+b)}$$

$$\eta_l \rightarrow \eta'_l = \eta - \xi_l \frac{\ln \rho}{a+b}, \quad \text{where} \quad \xi = \frac{\eta}{\eta_{\max}}$$

observable parametrization at one-emission level: $v = (k_t^2/Q^2)^a \exp(-b\eta)$

- NLL precision requires scaling to be maintained after additional emissions

Logarithmic accuracy – Analytic proof

- Lorentz transformation defined by shift $\tilde{K} \rightarrow K$

$$K^\mu = \tilde{K}^\mu - X^\mu, \quad \text{where} \quad X^\mu = p_j^\mu - (1-z)\tilde{p}_i^\mu$$

- X is small, but is it small enough? Rewrite

$$\Lambda_\nu^\mu(K, \tilde{K}) = g_\nu^\mu + \tilde{K}^\mu A_\nu + X^\mu B_\nu$$

- In NLL limit, coefficients scale as

$$A^\nu \xrightarrow{\rho \rightarrow 0} 2 \frac{\tilde{K} X}{\tilde{K}^2} \frac{\tilde{K}^\nu}{\tilde{K}^2} - \frac{X^\nu}{\tilde{K}^2}, \quad \text{and} \quad B^\nu \xrightarrow{\rho \rightarrow 0} \frac{\tilde{K}^\nu}{\tilde{K}^2}.$$

- Simplify situation by taking $a = 1, b = 0$ (worst offenders)

Relative momentum shift of soft emission particle l becomes

$$\Delta p_l^{0,3} / \tilde{p}_l^{0,3} \sim \rho^{1-\max(\xi_i, \xi_j)} \xrightarrow{\rho \rightarrow 0} 0$$

$$\Delta p_l^{1,2} / \tilde{p}_l^{1,2} \sim \rho^{1-\xi_i} \xrightarrow{\rho \rightarrow 0} 0$$

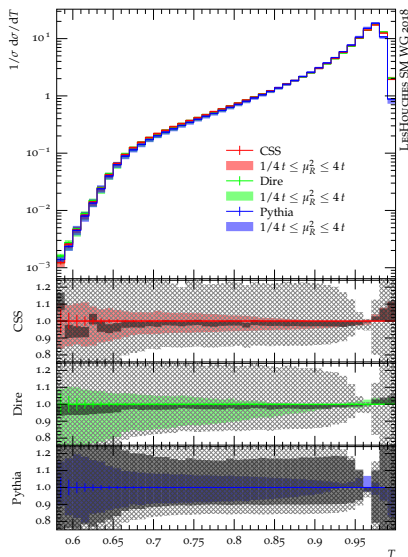
- For hard momenta, leading terms in X^μ cancel exactly
Remaining components scale as ρ or stronger

The elusive parton-shower uncertainty

Scale variations

[LesHouches] arXiv:1605.04692, arXiv:1803.07977

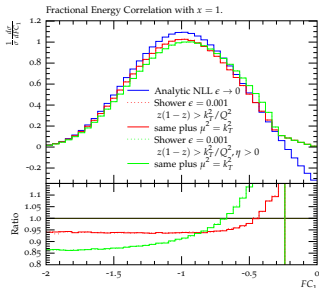
- First systematic attempt to estimate PS variations by MCnet groups at LesHouches 2015/2017 →
- Renormalization scale uncertainties based on order α_s^2 corrections to soft enhanced part of kernels
- Kinematics and evolution variable remain similar among contenders



Momentum conservation

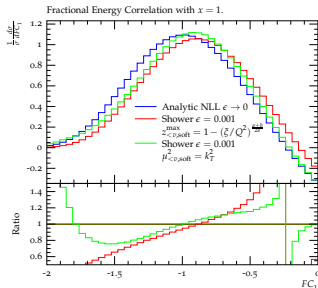
[Reichelt,Siegert,SH] arXiv:1711.03497

- Sizeable differences between NLL & PS away from $s \rightarrow \infty$ limit
- “guesstimate” of uncertainty from momentum mapping
NLP resummation needed to improve systematically



Single emission effects

- 4-mom conservation
- PS sectorization
- k_T scale in coll. terms



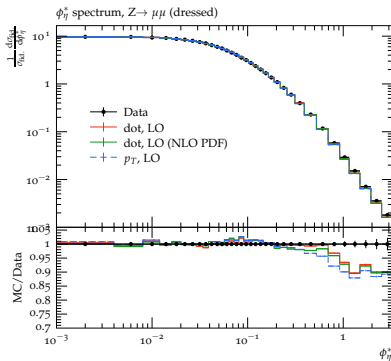
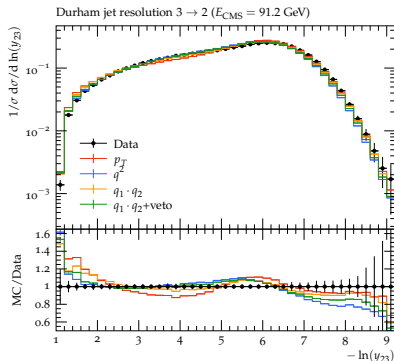
Multiple emission effects

- z bounds by unitarity
- k_T scale by unitarity

Herwig angular ordered parton showers

[Bewick,Ferrario-Ravasio,Richardson,Seymour] arXiv:1904.11866, arXiv:2107.04051

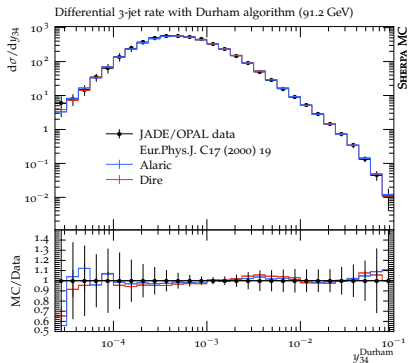
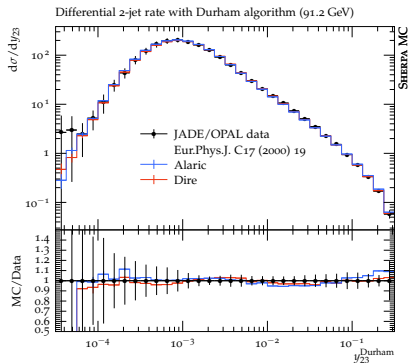
■ Comparison of q_T , q^2 & dot product preserving recoil schemes



Alaric parton shower

[Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057

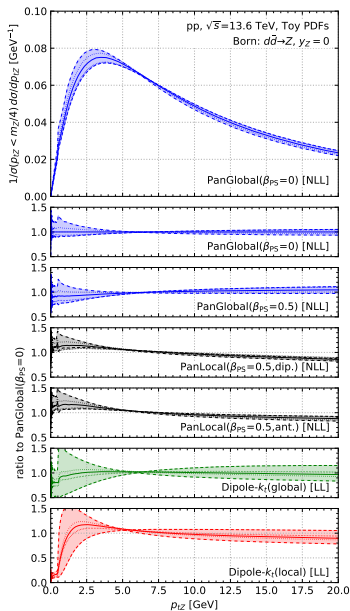
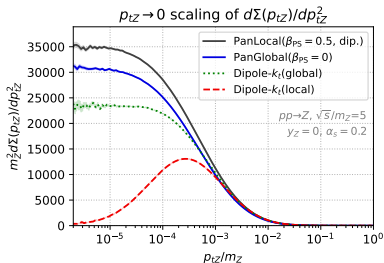
■ Comparison to experimental data from LEP



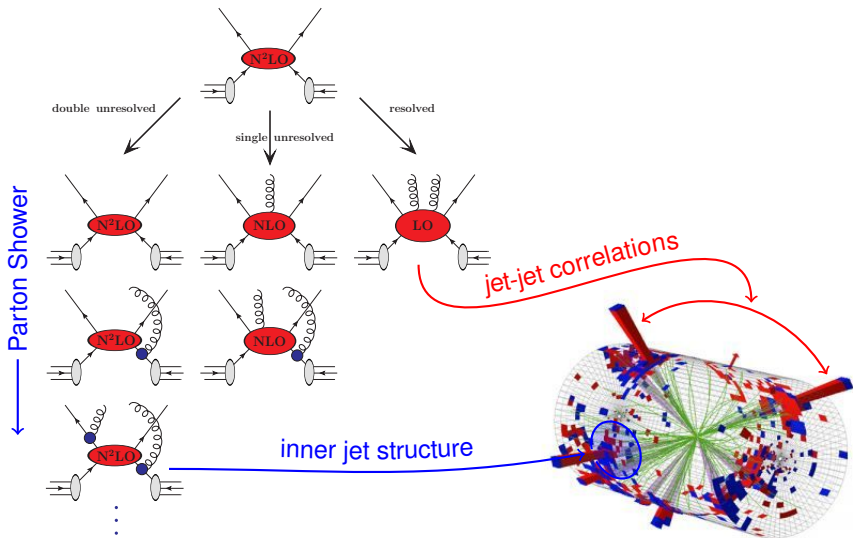
PanScales parton shower

[van Beekveld, Ferrario Ravasio, Hamilton, Salam, Soto-Ontoso, Soye, Verheyen] arXiv:2207.09467

- Comparison of different PanScales showers all provably NLL accurate
- Toy PDF, fixed flavor initial state
- Conventional dipole schemes do not reproduce [Parisi, Petronzio] NPB154(1979)427



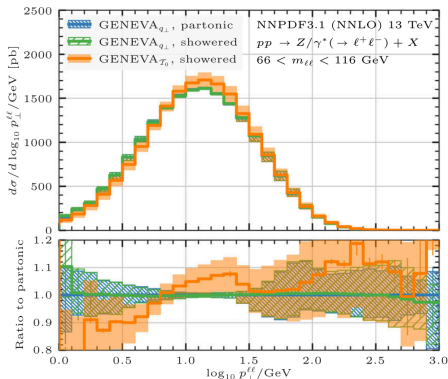
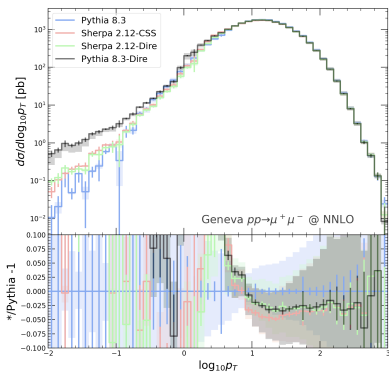
Impact of parton-shower variations on matching



Impact of parton-shower variations on matching

[D. Napoletano, HP2 2022], [Alioli et al.] arXiv:2102.08390

■ NNLO+PS precise predictions for $pp \rightarrow Z$ from Geneva



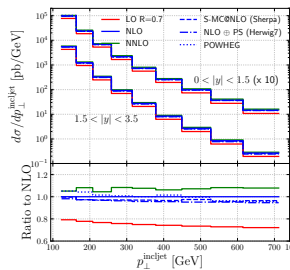
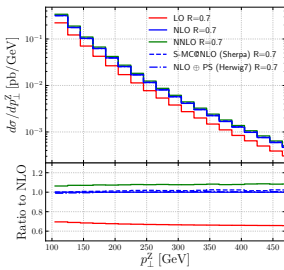
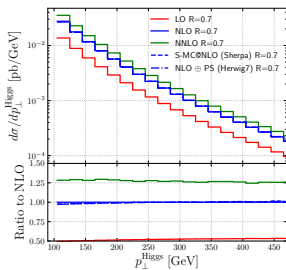
■ Parton shower scheme uncertainty

■ Choice of resolution variable

Impact of parton-shower variations on matching

[Bellm et al.] arXiv:1903.12563

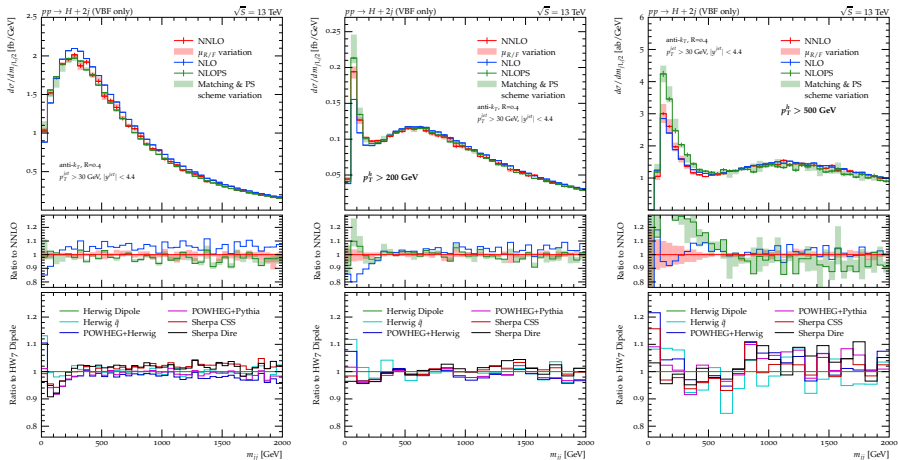
- Inclusive p_{\perp} spectra for different processes
- Comparison between different MC (Herwig, Sherpa, POWHEG+Herwig)



Impact of parton-shower variations on matching

[Buckley et al.] arXiv:2105.11399

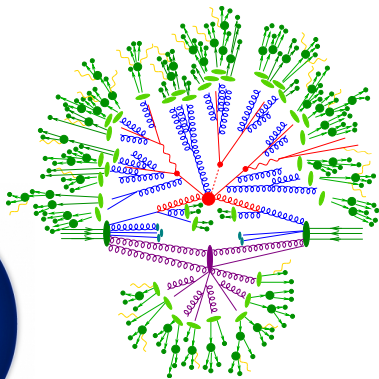
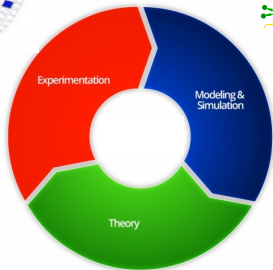
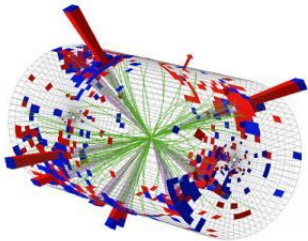
■ m_{jj} of two leading jets in VBF Higgs production



Summary and Outlook

- Lots of activity in parton shower development ...
 - Logarithmic precision [PanScales,Deductor,Herwig,Sherpa,...]
 - Higher-order kernels [Vincia,Sherpa,Herwig,...]
 - Interplay w/ NNLL, CMW [PanScales,Sherpa,...]
- ... and matching to fixed-order calculations
 - Improvements at NLO [Herwig,Pythia,Sherpa,...]
 - Resummation based [Geneva,MINNLO_{PS}]
 - Fully differential [Vincia,UN^XLOPS,TOMTE]
- Still, many questions remain [Campbell et al.] arXiv:2203.11110
 - Systematic treatment of power corrections
 - Massive quark production & evolution
 - Interplay with hadronization
 - ...

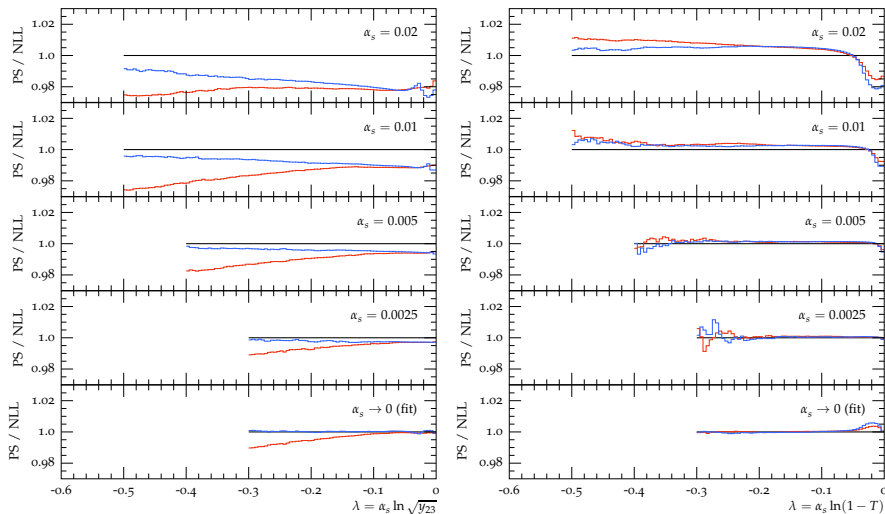
Exciting times ahead!



$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\Psi}\not{D}\Psi + h.c.$$

Accuracy of Alaric scheme – Numerical checks

[Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057



- At fixed $\lambda = \alpha_s \log v$, deviation from NLL should be proportional to α_s
- Dire algorithm (red) fails, Alaric (blue) passes

Comparison to experimental data: LEP I

[Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057

■ Comparison to experimental data from LEP

