Precision Event Generation for the LHC

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CP^3 - Origins Seminar

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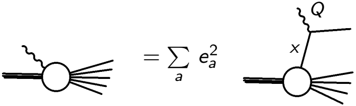
QCD in the context of collider experiments

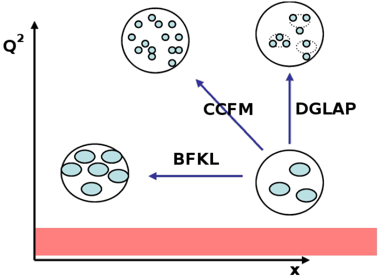
Ansatz: hadronization is universal and occurs at $Q^2 \approx \mathcal{O}(\Lambda^2_{QCD})$
described by parton distributions (PDF) and fragmentation functions (FF)
$\Rightarrow$ factorization formula for hadronic cross section in DIS

$$\sigma = \sum_a \int \mathrm{d}x\, f_a(x, Q^2)\, \mathrm{d}\hat{\sigma}_a(x, Q^2)$$

$f_a(x, Q^2)$ - PDF probability to extract parton a with energy fraction x from initial hadron at scale Q^2

$\hat{\sigma}_a(x, Q^2)$ - partonic cross section





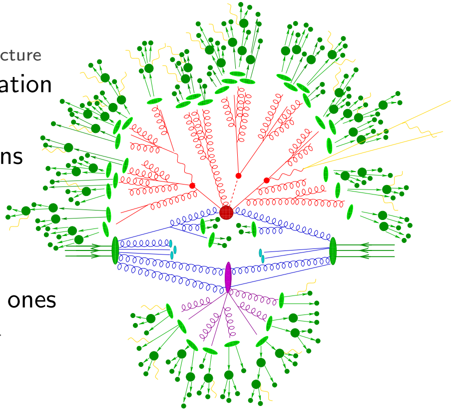
Energy increase $\Rightarrow$ extra partons
Emission phase space separates into

- DGLAP regime Q^2-evolution
- BFKL regime $1/x$-evolution

CCFM "interpolates" between DGLAP and BFKL

Event generation in standard Monte Carlo

1. Matrix Element (ME) generators red blobs
   simulate "central" part of the event
2. Parton Showers (PS) red & blue tree structure
   produce additional "hard" QCD radiation
3. Multiple interaction models purple blob
   simulate "secondary hard" interactions
4. Fragmentation models light green blobs
   hadronize QCD partons
5. Hadron decay modules dark green blobs
   decay primary hadrons into observed ones
6. Photon emission generators yellow stuff
   simulate additional QED radiation



Focus on ME, PS and their interplay in this talk

Find more in recently published review [Buckley et al.] Phys.Rept.504(2011)145

Matrix elements

Event generation starts with the computation of $\hat{\sigma}$

$$\hat{\sigma}_a = \int \mathrm{d}\Phi_n \frac{\mathrm{d}\hat{\sigma}_a}{\mathrm{d}\Phi_n}$$

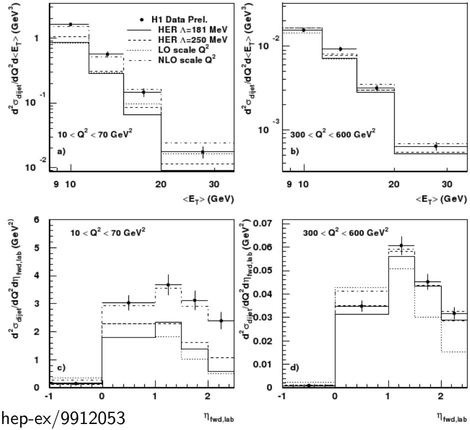
What's the tricky point of this formula ?

- Differential cross section $\mathrm{d}\hat{\sigma}_a/\mathrm{d}\Phi_n$
  difficult to obtain for large number of particles n
  solved problem at tree-level, newly emerging techniques at 1-loop
- $\mathrm{d}\Phi_n$ implies high-dimensional integral
  $\mathrm{d}\hat{\sigma}_a/\mathrm{d}\Phi_n$ most commonly sharply peaked
  much more problematic than ME generation because
  suitable phase space mappings must be found for arbitrary ME
- "Internal" degrees of freedom helicity & color
  num. effort for sum is (naively) $\sim 2^n$ / $3^n - 8^n$
  sampling can be much more efficient, depending on n

Simple solution: Employ leading-order ME and produce more partons with PS

Traditional "LO$\otimes$PS" method, around for $\sim$20-30 years

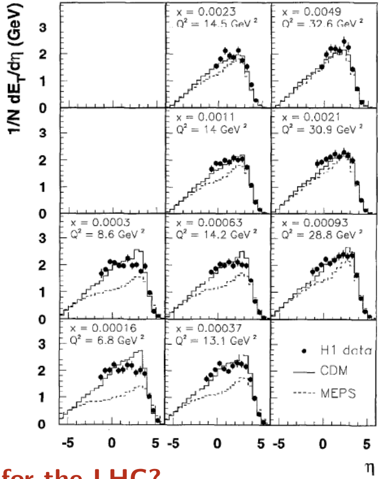
Trying the simple solution in Deep Inelastic Scattering



hep-ex/9912053

Example: Energy flow PLB356(1995)118

- MEPS (DGLAP) fails for low Q^2
- CDM partially agrees with data



Example: Di-jet events EPJC19(2001)289

- PS fails to describe jet spectra
- Low-Q^2 region especially problematic

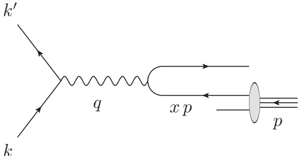
Should we really rely on such simulations for the LHC?

Why the simple solution does not work

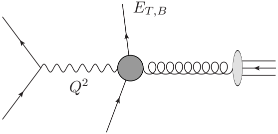
Leading order $e^{\pm}p$ - scattering in collinear factorization Breit frame

- **No transverse momentum**
  No QCD activity, i.e. no jets!
- Kinematical variables

$$Q^2 = q^2 = (k' - k)^2 \text{ and } x = \frac{Q^2}{2\,q \cdot p}$$



What happens at higher orders ?



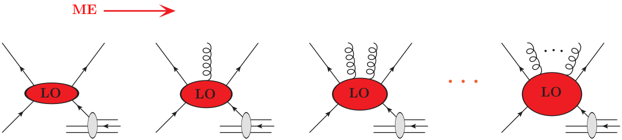
- **Transverse momentum** $E^2_{T,B}$
- $E^2_{T,B} \lesssim Q^2 \Leftrightarrow e^{\pm}q \to e^{\pm}q$ 0-jet like
  $Q^2 \lesssim E^2_{T,B} \Leftrightarrow \gamma^* g \to jets$ 2-jet like

On average Q^2 of the exchanged photon tends to be zero

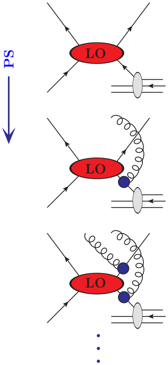
$\to$ 2-jet like situation most common, but badly described by LO⊗PS
PS is suited for intra-jet evolution, not jet production

How to improve the simple picture

High-multiplicity LO matrix elements

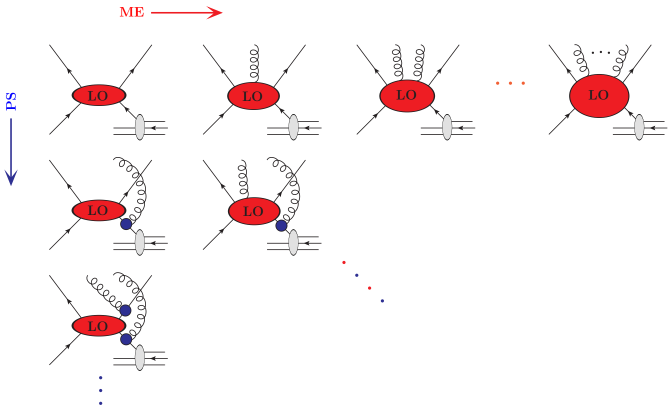


Novel parton showers



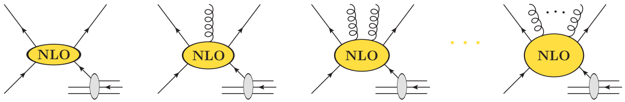
How to improve the simple picture

Matrix-element parton-shower merging (ME⊗PS)



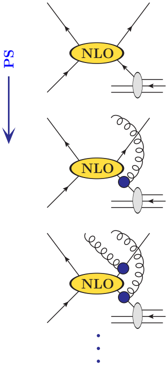
How to improve the simple picture

(High-multiplicity) NLO matrix elements



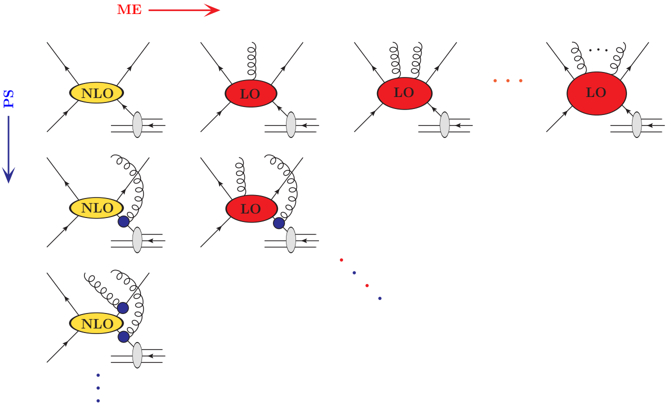
How to improve the simple picture

NLO matrix-element parton-shower matching



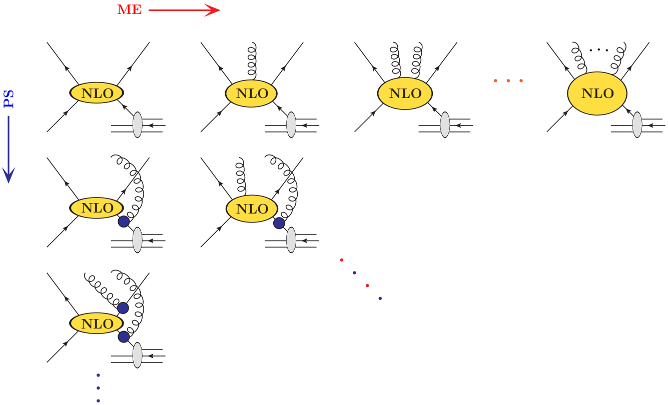
How to improve the simple picture

ME+PS merging with NLO "core" process (MEnloPS)



Where we are heading ...

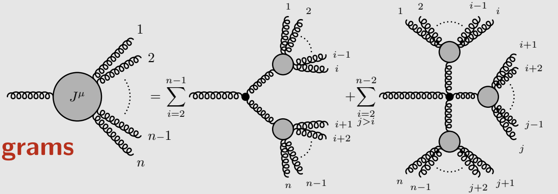
ME+PS merging with multiple NLO processes



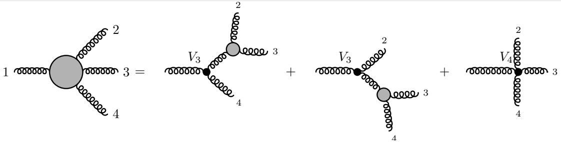
The quest for many jets

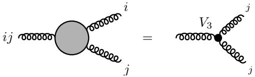
Example: Matrix-element generation in n-gluon process

- ME is sum of all possible Feynman graphs
- **Use recursive equation to find contributing diagrams**



Example: Diagrams for $g(1)g(2) \to g(3)g(4)$



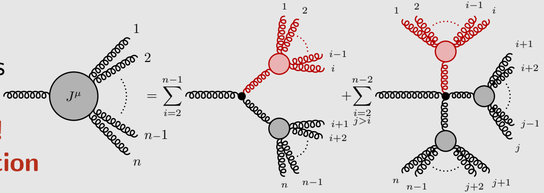


→ Translate diagrams into “helicity amplitudes” [Ballestrero,Maina] PLB350(1995)225
using the fact that L/R spinors are basic representations of the Lorentz group
→ Any scattering amplitude can be expressed in terms of spinor products (“almost” Lorentz invariants)

Note: No more than strings of up to four spinor products needed
alleviates computation due to reduced number of building blocks [Dixon] hep-ph/9601359

Example: ME generation in n-gluon process [Berends,Giele] NPB306(1988)759

- ME built from off-shell currents joined by vertices
  off-shell current ↔ internal
- **Read from right to left!**
  no redundant computation



Example: Computing $g(1)g(2) \to g(3)g(4)$

Step 1	$J_1 = \varepsilon(1)$	$J_2 = \varepsilon(2)$	$J_3 = \varepsilon(3)$	$J_4 = \varepsilon(4)$
Step 2	J_{12}	J_{13}	J_{23}	
Step 3	J_{123}			
Step 4	$A(1,2,3,4) = J_4^* J_{123}$			

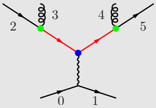
Performance of color dressed Berends-Giele recursion [Gleisberg,SH] JHEP08(2006)062

gg → ng	Cross section [pb]				
n $\sqrt{s}$ [GeV]	8 1500	9 2000	10 2500	11 3500	12 5000
Comix	0.755(3)	0.305(2)	0.101(7)	0.057(5)	0.026(1)
PRD67(2003)014026	0.70(4)	0.30(2)	0.097(6)		
NPB539(1999)215	0.719(19)				

The quest for many jets

Commonly used method to evaluate multi-particle phase space

- Guess peak structure of integrand from dynamics [Byckling,Kajantie] NPB9(1969)568



$\leftrightarrow$ $D_{iso}(23,45) \otimes P_0(23) \otimes P_0(45) \otimes D_{iso}(2,3) \otimes D_{iso}(4,5)$

- Combine channels corresponding to single diagrams into multi-channel and optimize [Kleiss,Pittau] CPC83(1994)141
- Refine single integration channels with VEGAS [Lepage] JCP27(1978)192

Only propagators and vertices introduce integration channels !

$\rightarrow$ phase space generation technique corresponds to ME generation diagrammatic and recursive way!

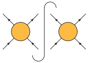
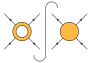
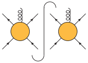
Other, less optimized / less general techniques exist, like Rambo & HAAG

Correlations and interferences within ME often render optimization cumbersome
Color- and/or helicity sampling impairs convergence

Most general-purpose phase-space generators employ above technique
AMEGIC++, Comix, MADEVENT, PHEGAS, WHIZARD, ...

How to construct an NLO MC

NLO calculation

- Born term: $B =$ 
- Virtual terms: $V = \sum 2\,\mathrm{Re}\{$  $\}$
- Real terms: $R = \sum$ 

V and R separately infrared divergent, but poles cancel

Singularities must be removed before MC-integration, usually subtracted

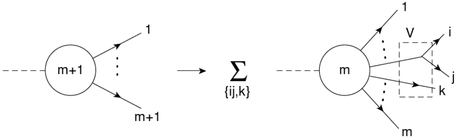
$$\sigma^{NLO} = \int \mathrm{d}\Phi_B \left(\mathrm{B} + \tilde{\mathrm{V}}\right) + \int \mathrm{d}\Phi_R\, \mathrm{R} = \int \mathrm{d}\Phi_B \left[\left(\mathrm{B} + \tilde{\mathrm{V}} + \mathrm{I}\right) + \int \mathrm{d}\Phi_{R|B}\, (\mathrm{R} - \mathrm{S})\right]$$

S - subtraction term constructed such that IR singularities in R are removed

I - integrated subtraction term locally (in Φ_B) compensates $\mathrm{S} \rightarrow 0 \stackrel{!}{=} \mathrm{I} - \int \mathrm{d}\Phi_{R|B}\, \mathrm{S}$

Catani-Seymour dipole method [Catani,Seymour] NPB485(1997)291

Schematically: $\mathrm{S} \rightarrow \sum_{\{i,j\}} \sum_{k \neq i,j} \mathrm{D}^{(\mathrm{S})}_{ij,k}$



Summing the logs

With $\{\vec{a}\}$ a set of partons flavors $\{\vec{f}\}$, momenta $\{\vec{p}\}$

$\rightarrow$ **Real-emission contribution to NLO cross section**

$$\mathrm{d}\sigma_R(\{\vec{p}\}) = \sum_{\{\vec{f}\}} \mathrm{d}\sigma_R(\{\vec{a}\}) \qquad \mathrm{d}\sigma_R(\{\vec{a}\}) = \mathrm{d}\Phi_R(\{\vec{p}\})\,\mathrm{R}(\{\vec{a}\})$$

where $\mathrm{R}(\{\vec{a}\}) = \mathcal{L}(\{\vec{a}\})\,\mathcal{R}(\{\vec{a}\})$ and $\mathcal{L}(\{\vec{a}\};\mu^2) = x_1 f_{f_1}(x_1,\mu^2)\, x_2 f_{f_2}(x_2,\mu^2)$
$\mathrm{d}\Phi_R$ contains initial-state phase space $\mathrm{d}\log x_1\, \mathrm{d}\log x_2$
$\mathcal{R}(\{\vec{a}\}) = |\mathcal{M}_R|^2(\{\vec{a}\})/[F(\{\vec{a}\})S(\{\vec{f}\})]$ with symmetry factor S, flux F

Similar formulas for Born-level term $\mathrm{B}(\{\vec{a}\})$ one parton less, of course

Assume generalized "dipole terms", such that think of $\mathrm{D}^{(\mathrm{S})}_{ij,k}$ on previous slide

$$\mathcal{R}(\{\vec{a}\}) \xrightarrow{\text{soft/collinear}} \sum_{\{i,j\}}\sum_{k\neq i,j} \mathcal{D}^{(\mathrm{S})}_{ij,k}(\{\vec{a}\})$$

Define partition of real-emission term $\mathcal{R}(\{\vec{a}\}) = \sum_{\{i,j\}}\sum_{k\neq i,j}\mathcal{R}_{ij,k}(\{\vec{a}\})$

$$\mathcal{R}_{ij,k}(\{\vec{a}\}) := \rho_{ij,k}(\{\vec{a}\})\,\mathcal{R}(\{\vec{a}\})\,, \quad \text{where} \quad \rho_{ij,k}(\{\vec{a}\}) = \frac{\mathcal{D}^{(\mathrm{S})}_{ij,k}(\{\vec{a}\})}{\sum_{\{m,n\}}\sum_{l\neq m,n}\mathcal{D}^{(\mathrm{S})}_{mn,l}(\{\vec{a}\})}$$

Summing the logs

$\mathcal{D}^{(\mathrm{S})}_{ij,k}(\{\vec{a}\})$ defines parton maps think of Catani-Seymour dipoles

$$b_{ij,k}(\{\vec{a}\}) = \begin{cases} \{\vec{f}\}\setminus\{f_i,f_j\}\cup\{f_{\widetilde{ij}}\} \\ \{\vec{p}\}\to\{\vec{\tilde{p}}\} \end{cases} \quad\leftrightarrow\quad r_{\widetilde{ij},\tilde{k}}(f_i,\Phi_{R|B}\,;\{\vec{a}\}) = \begin{cases} \{\vec{f}\}\setminus\{f_{\widetilde{ij}}\}\cup\{f_i,f_j\} \\ \{\vec{\tilde{p}}\}\to\{\vec{p}\} \end{cases}$$

- $b_{ij,k}$ converts real-emission configuration to Born-level
- $r_{\widetilde{ij},\tilde{k}}$ converts Born-level to real-emission needs extra flavor & phase space

Trivially factorize real-emission term into Born and radiative contribution

$$\mathrm{d}\sigma_R(\{\vec{a}\}) = \sum_{\{i,j\}}\sum_{k\neq i,j} \mathrm{d}\sigma_B(b_{ij,k}(\{\vec{a}\}))\,\mathrm{dP}_{ij,k}(\{\vec{a}\})$$

differential emission probability is $\mathrm{dP}_{ij,k}(\{\vec{a}\}) = \mathrm{d}\Phi^{ij,k}_{R|B}(\{\vec{p}\})\,\dfrac{\mathrm{R}_{ij,k}(\{\vec{a}\})}{\mathrm{B}(b_{ij,k}(\{\vec{a}\}))}$

Subtraction algorithms predict $\mathrm{dP}_{ij,k}$ in the soft/collinear limits via

$$\mathcal{D}^{(\mathrm{S})}_{ij,k}(\{\vec{a}\}) \xrightarrow{\text{soft/collinear}} \frac{S(b_{ij,k}(\{\vec{f}\}))}{S(\{\vec{f}\})}\,\frac{1}{2\,p_ip_j}\,8\pi\,\alpha_s\,\mathcal{B}(b_{ij,k}(\{\vec{a}\}))\otimes V_{ij,k}(p_i,p_j,p_k)\;,$$

Note the symmetry factors $\leftrightarrow$ factorization of invariant ME, not of specific process

$\otimes\to$ spin & color-correlations between $\mathcal{B}$ and V

Summing the logs

Now make an approximation replace correlated with uncorrelated dipole kernel

$$\mathcal{B}(b_{ij,k}(\{\vec{a}\})) \otimes V_{ij,k}(p_i, p_j, p_k) \to \mathcal{B}(b_{ij,k}(\{\vec{a}\}))\, \mathcal{K}_{ij,k}(p_i, p_j, p_k)$$

Parametrize radiative phase space: $\mathrm{d}\Phi^{ij,k}_{R|B}(\{\vec{p}\}) = \frac{1}{16\pi^2}\,\mathrm{d}t\,\mathrm{d}z\,\frac{\mathrm{d}\phi}{2\pi}\,J_{ij,k}(t,z,\phi)$

Assume phase space gets filled successively in $t \leftrightarrow$ partons can be distinguished

Must adapt symmetry factors: $\frac{S(b_{ij,k}(\{\vec{f}\}))}{S(\{\vec{f}\})} \to \frac{1}{S_{ij}} = \begin{cases} 1/2 & \text{if } i,j > 2,\ b_i = b_j \\ 1 & \text{else} \end{cases}$

Combining everything gives differential radiation probability

$$\mathrm{dP}^{(\mathrm{PS})}_{ij,k}(\{\vec{a}\}) = \frac{\mathrm{d}t}{t}\,\mathrm{d}z\,\frac{\mathrm{d}\phi}{2\pi}\,\frac{\alpha_s}{2\pi}\,\frac{1}{S_{ij}}\,J_{ij,k}(t,z,\phi)\,\mathcal{K}_{ij,k}(t,z,\phi)\,\frac{\mathcal{L}(\{\vec{a}\};t)}{\mathcal{L}(b_{ij,k}(\{\vec{a}\});t)}$$

Iterate this equation for higher-multi ME

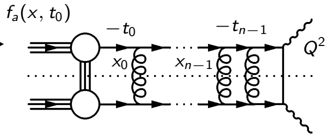
$\to$ ladder-like structure of amplitude squared

with strong ordering in scales $t_0 < \ldots < t_n$

Factorization at any stage above Λ_{QCD}

can split emissions off ME one by one

$\leftrightarrow$



Corrections induced by $\mathrm{dP}^{(\mathrm{PS})}_{ij,k}$ can be large and must be resummed

In inclusive case $t \in [0, \infty)$ divergences in $\mathcal{K}_{ij,k}$ cancel ε-poles in $\mathrm{V} \to$ unitarity !

Summing the logs

→ **No-emission probability** from Poisson statistics implementing unitarity constraint

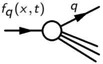
$$\mathcal{P}^{(\mathrm{PS})}_{\widetilde{ij},\tilde{k}}(t',t'';\{\vec{a}\}) = \exp\left\{ -\sum_{f_i=q,g} \int_{t'}^{t''} \int_{z_{\min}}^{z_{\max}} \int_0^{2\pi} \mathrm{dP}^{(\mathrm{PS})}_{ij,k}(r_{\widetilde{ij},\tilde{k}}(\{\vec{a}\})) \right\} .$$

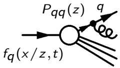
Note: $r_{\widetilde{ij}\tilde{k}}$ implicitly and uniquely defined by subtraction scheme, i.e. $\mathcal{K}_{ij,k}$

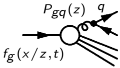
Assume IF-splitting → Lumi ratio $\frac{x}{z} f_{f_i}(\frac{x}{z},t)/x f_{f_{\widetilde{ij}}}(x,t)$, symmetry factor 1

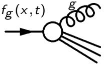
$$\frac{\partial \log \mathcal{P}^{(\mathrm{PS})}_{\widetilde{ij},\tilde{k}}(t,t';\{\vec{a}\})}{\partial \log(t/\mu^2)} = \int_x^{z_{\max}} \frac{\mathrm{d}z}{z} \int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi} \sum_{f_i=q,g} \frac{\alpha_s}{2\pi} J_{ij,k}(t,z,\phi)\, \mathcal{K}_{ij,k}(t,z,\phi)\, \frac{f_{f_i}(\frac{x}{z},t)}{f_{f_{\widetilde{ij}}}(x,t)}$$

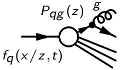
Voilà, the DGLAP equation ! imagine $J_{ij,k}(t,z,\phi)\mathcal{K}_{ij,k}(t,z,\phi) \to P_{i\,\widetilde{ij}}(z)$

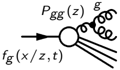
$\frac{\mathrm{d}}{\mathrm{d}\log(t/\mu^2)}$ 

$= \int_x^1 \frac{\mathrm{d}z}{z} \frac{\alpha_s}{2\pi}$ 

$+ \int_x^1 \frac{\mathrm{d}z}{z} \frac{\alpha_s}{2\pi}$ 

$\frac{\mathrm{d}}{\mathrm{d}\log(t/\mu^2)}$ 

$= \sum_{i=1}^{2n_f} \int_x^1 \frac{\mathrm{d}z}{z} \frac{\alpha_s}{2\pi}$ 

$+ \int_x^1 \frac{\mathrm{d}z}{z} \frac{\alpha_s}{2\pi}$ 

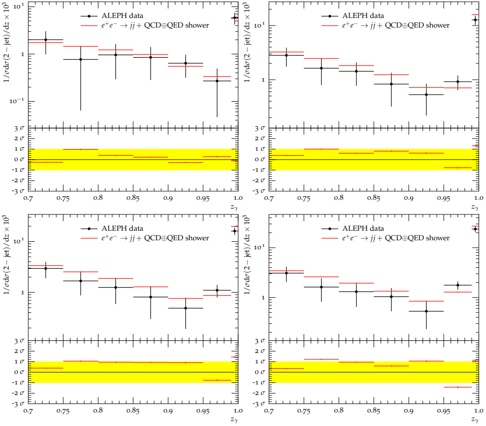
The Monte-Carlo implementation of this is a modern parton shower:

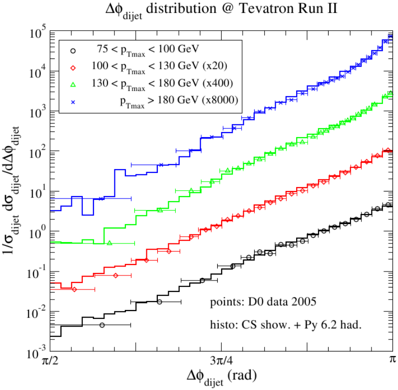
- Generate emission scale t from parton a at t' via

$$1-\mathcal{P}^{(PS)}_{\widetilde{ij},\tilde{k}}(t,t';\{\vec{a}\}) \stackrel{!}{=} \# \,, \qquad \text{where} \qquad \# \in [0,1] \text{ random}$$

- Generate splitting variable z, angle ϕ and flavor f_i according to $\mathrm{dP}^{(\mathrm{PS})}_{ij,k}(\{\vec{a}\})$

γ-fragmentation function PRD81(2010)034026

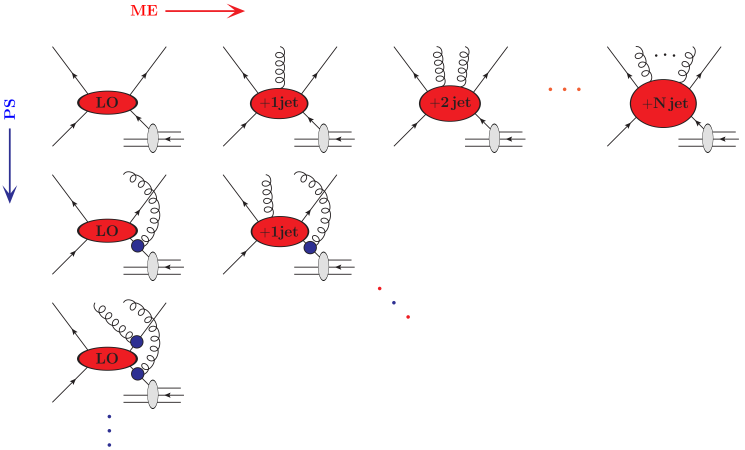




[Schumann,Krauss] JHEP03(2008)038

Fixed order vs. resummation: Part I - ME⊗PS

Chance and challenge: ME and PS can simulate the same thing!



Basic idea: **Use ME/PS in regime of their respective strengths**

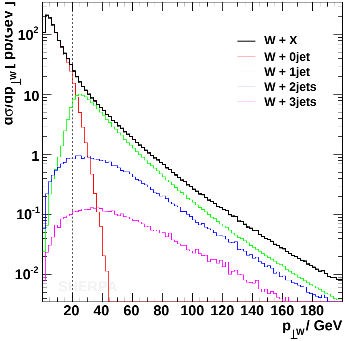
Fixed order vs. resummation: Part I - ME$\otimes$PS

Separate phase space into "hard" and "soft" region

[Catani,Krauss,Kuhn,Webber] JHEP11(2001)063
[Krauss,Schumann,Siegert,SH] JHEP05(2009)053

- Matrix elements populate hard domain
- Parton shower populates soft domain

need criterion to define "hard" & "soft"
as above and below a certain cut $\longleftrightarrow$
$\rightarrow$ **Jet criterion** Q e.g. k_T-jet measure



Formally: replace kernels in PS evolution with note that $\mathcal{K}_{ij,k} = \mathcal{K}^{\rm ME}_{ij,k} + \mathcal{K}^{\rm PS}_{ij,k}$

$$\mathcal{K}^{\rm ME}_{ij,k}(t,z,\phi) = \mathcal{K}_{ij,k}(t,z,\phi)\,\Theta\left[Q_{ij,k}(t,z,\phi) - Q_{\rm cut}\right] \quad\rightarrow\quad \mathcal{P}^{\rm (PS)\,ME}_{\widetilde{ij},\tilde{k}}(t,t')$$

$$\mathcal{K}^{\rm PS}_{ij,k}(t,z,\phi) = \mathcal{K}_{ij,k}(t,z,\phi)\,\Theta\left[Q_{\rm cut} - Q_{ij,k}(t,z,\phi)\right] \quad\rightarrow\quad \mathcal{P}^{\rm (PS)\,PS}_{\widetilde{ij},\tilde{k}}(t,t')$$

Correct PS **emission** probability by $\mathrm{dP}_{ij,k}(\{\vec{a}\})\,/\,\mathrm{dP}^{\rm (PS)}_{ij,k}(\{\vec{a}\})$ in ME domain

Fixed order vs. resummation: Part I - ME$\otimes$PS

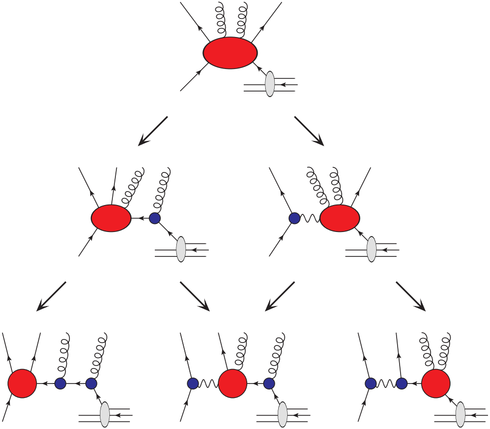
Matrix elements can have very different PS equivalents depending on kinematics

Must reduce full high-multi ME to either of these configurations in order to start PS

Radiation effects off intermediate legs must be modeled to account for Sudakov suppression $\rightarrow$ truncated PS

Method: PRD57(1998)5767, JHEP05(2009)053

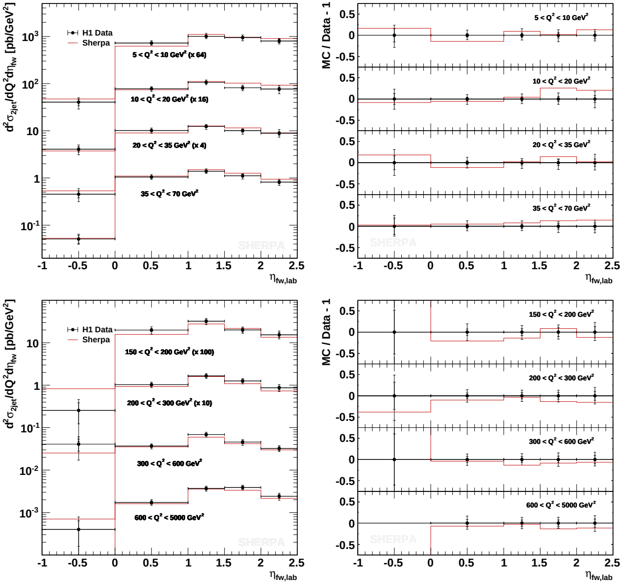
- Probability to identify splitting given by PS's branching eqns
- Reduced ME configuration defined by "inverted" PS splitting kinematics
- **Continue until** $2 \rightarrow 2$ **"core"**



Core processes set the hardness scale of events $\rightarrow \mu_F$

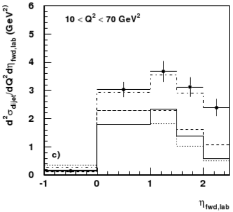
i.e. no scale should be larger than this PRD70(2004)114009, JHEP05(2009)053

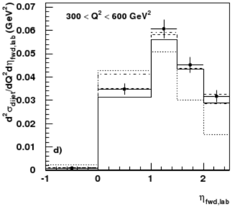
ME⊗PS at work: DIS



Compare to LO⊗PS results

→ MC status 1999 vs. 2010

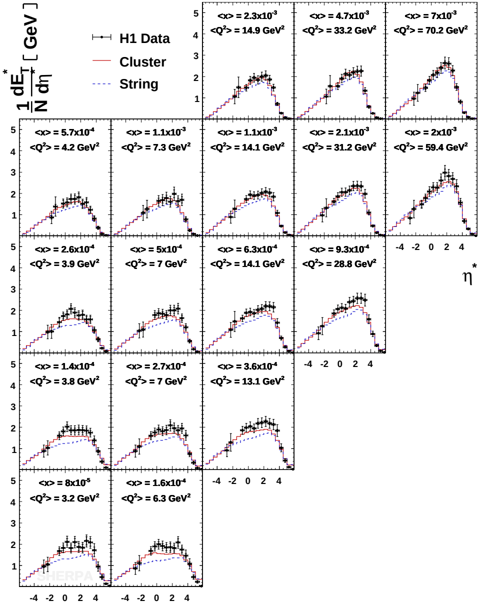




[Carli,Gehrmann,SH]

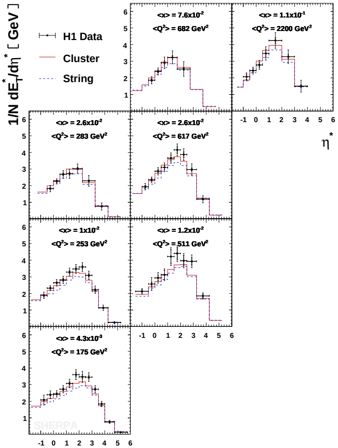
EPJC67(2010)73

ME⊗PS at work: DIS



Transverse energy flow

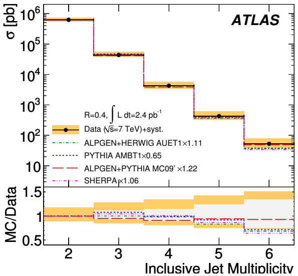
Hadronization effects often large but overall picture much improved

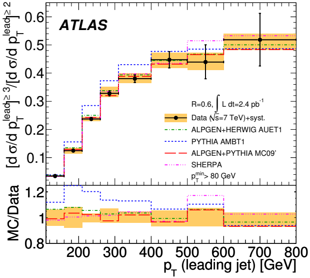


[Carli,Gehrmann,SH] EPJC67(2010)73

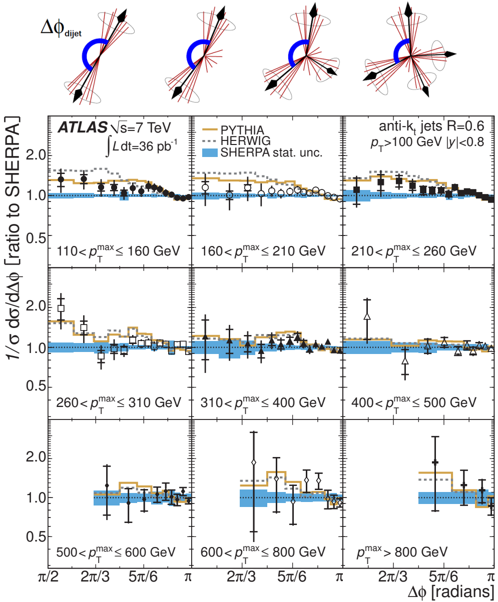
ME⊗PS at work: Jet production

Multi-jet rates





[ATLAS] arXiv:1107.2092 [hep-ex]

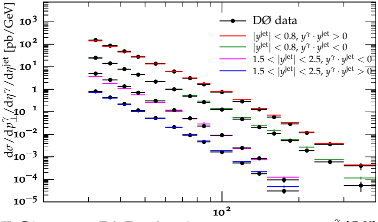


[ATLAS] arXiv:1102.2696

ME⊗PS at work: Prompt photons

Photon p_T spectra PLB666(2008)435

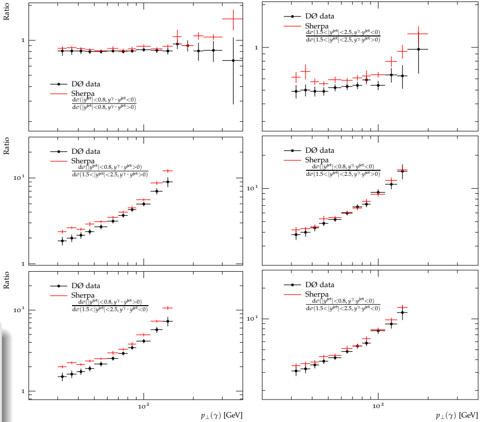
in regions of jet rapidity/orientation
scaled by 5, 1, 0.3 and 0.1 top to bottom



F.Siegert, PhD thesis

Ratio of photon p_T spectra PLB666(2008)435

compare regions of jet rapidity/orientation



F.Siegert, PhD thesis

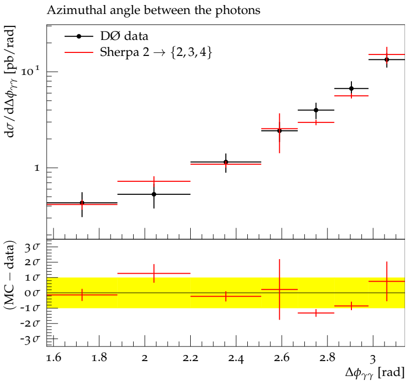
"Democratic" model ZPC62(1994)311

- Treat partons and γ equally
- Combine ME of various parton/γ multiplicity with
- Interleaved QCD⊕QED PS

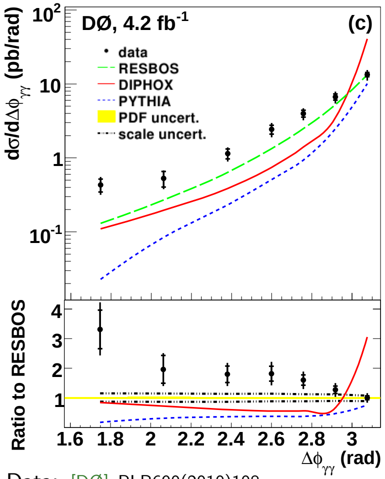
[Schumann,Siegert,SH] PRD81(2010)034026

ME⊗PS at work: Di-photon production

$E_T^{\gamma 1} > 21$ GeV, $E_T^{\gamma 1} > 20$ GeV,
$|\eta^\gamma| < 0.9$, $E_T^{R=0.4} - E_T^\gamma < 2.5$ GeV



[Schumann,Siegert,SH] PRD81(2010)034026



Data: [DØ] PLB690(2010)108

SHERPA prediction: Merged 2→{2+3+4}-jet/γ plus $gg \to \gamma\gamma\, box$

On to higher precision

Improving predictions for hard scattering processes
requires efficient computation of QCD one-loop corrections

Idea: Share the workload between tree-level and loop-level programs

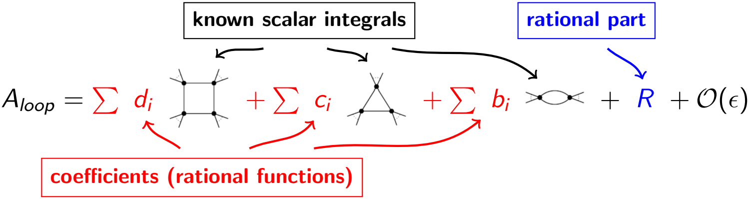
$$\sigma_{\rm NLO} = \int d\Phi_{\rm B} \sum \left\{ {\rm B} + \tilde{\rm V} + {\rm I} \right\} + \int d\Phi_R \sum \left\{ {\rm R} - {\rm S} \right\}$$

Standardized interface exists as Les Houches accord [Binoth et al.] CPC181(2010)1612

- One-Loop Engines (OLEs) like BlackHat [Berger et al.] PRD78(2008)036003
  or GOLEM [Binoth et al.] CPC180(2009)2317 provide virtual piece or more
- ME generator takes care of Born, real emission, subtraction
  phase-space integration and event generation

The quest for many jets reloaded

Use generalized unitarity to compute virtual corrections
Basic ideas: [Bern,Dixon,Dunbar,Kosower] NPB435(1995)59 NPB513(1998)3



Cut-constructible part of virtual amplitude reduced to scalar integrals
at integrand level → determine coefficients d, c & b from tree amplitudes
[Ossola,Papadopoulos,Pittau] NPB763(2007)147, [Forde] PRD75(2007)125019

Fully automated by BlackHat collaboration PRD78(2008)036003
[Berger,Bern,Dixon,Febres-Cordero,Gleisberg,Forde,Ita,Kosower,Maître], also [Diana,Ozeren,SH]

Fast trees (from N=4 SYM, BG or BCFW) recycled into fast loops
Rational piece from D-dimensional unitarity or loop-level recursion

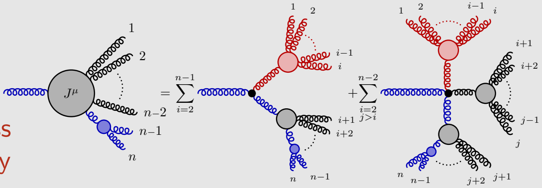
The quest for many jets reloaded

One-Loop-Engines attacks higher and higher jet multiplicity
→ need to compute real-radiation & infrared subtraction terms efficiently

Is Catani-Seymour dipole subtraction an obstacle for high multiplicity?

Tree-level problem ⇒ Extend CDBG recursion from [Duhr,Maltoni,SH] JHEP 08(2006)062

- Fix spectator parton as "final" leg in amplitude
- Recycle subamplitudes from real-radiation process and dipoles simultaneously



Use framework of existing ME generator Comix [Gleisberg,SH] JHEP 12(2008)039

- Real subtraction terms validated and optimized ✓
- Integrated subtraction terms validated, need optimization

No indication that CS method is bottleneck for automating NLO!
Number of real subtraction terms grows approximately like N^3 i.e. very mild

The quest for many jets reloaded

$e^+e^- \to$jets (91.2 GeV) real-radiation & subtraction only

σ_{R-S} [pb]	Number of jets				
n k_T-jets	2	3 $\alpha_c = 0.1$	4 $\alpha_c = 0.03$	5 $\alpha_c = 0.01$	6 $\alpha_c = 0.003$
AMEGIC++/BlackHat Comix	-357(2) -356(2)	1667(17) 1631(16)	1047(8) **1056(11)**	167(11) **169(2)**	– **12.01(6)**
Speedup*	0.63	0.52	**1.54**	**2.0**	–

$pp \to e^+e^-$+jets (7 TeV) real-radiation & subtraction only

σ_{R-S} [pb]	Number of jets					
n k_T-jets	0	1 $\alpha_c = 0.1$	2 $\alpha_c = 0.03$	3 $\alpha_c = 0.01$	4 $\alpha_c = 0.003$	5 $\alpha_c = 0.001$
AMEGIC++/BlackHat Comix	30.2(1) 30.0(1)	47.2(5) 46.1(5)	93.0(9) **92.6(6)**	55.8(3) **54.8(4)**	? **23.2(2)**	– 
Speedup*	0.4	0.46	**1.92**	**1.76**	?	–

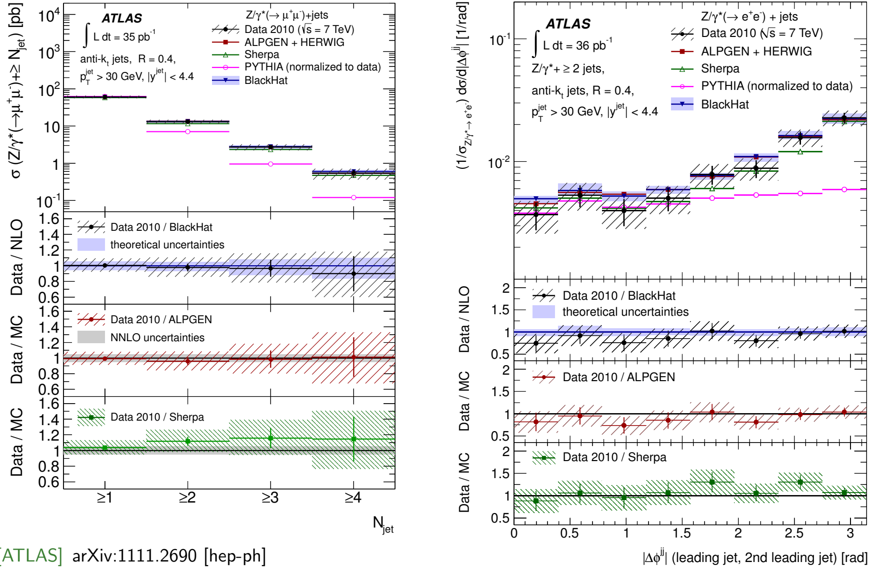
$pp \to e^+\nu_e$+jets (7 TeV) real-radiation & subtraction only

σ_{R-S} [pb]	Number of jets					
n k_T-jets	0	1 $\alpha_c = 0.1$	2 $\alpha_c = 0.03$	3 $\alpha_c = 0.01$	4 $\alpha_c = 0.003$	5 $\alpha_c = 0.001$
AMEGIC++/BlackHat Comix	-200(1) -198(1)	297(3) 297(3)	576(6) 586(6)	342(2) **343(1)**	? **143(1)**	– **31.7(6)**
Speedup*	0.4	0.9	0.6	**3.39**	?	–

*Timing is for complete integration (i.e. matrix-element *and* phase-space)

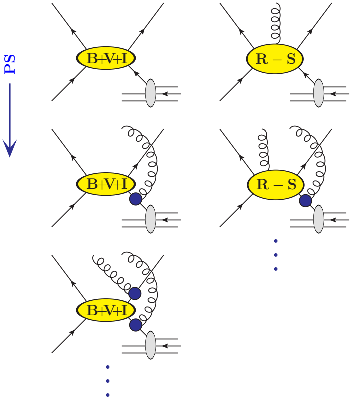
The quest for many jets reloaded

Synergy between OLE and MC allows to attack e.g. W+3/4 jets, Z+3/4 jets



[ATLAS] arXiv:1111.2690 [hep-ph]

NLO challenge: B-, V-, I- and S-terms kinematically different from R



Fixed order vs. resummation: Part II - NLO matching

Recover NLO-accurate radiation pattern in PS through correction weight

$$w_{ij,k}(\{\vec{a}\}) = \mathrm{dP}_{ij,k}(\{\vec{a}\})\,/\,\mathrm{dP}^{(\mathrm{PS})}_{ij,k}(\{\vec{a}\})$$

Easy to automate in general-purpose Monte-Carlo, all input is tree-level only

Approximate "seed cross section" using local K-factor $\bar{\mathrm{B}}/\mathrm{B}$

$$\frac{\bar{\mathrm{B}}(\{\vec{a}\})}{\mathrm{B}(\{\vec{a}\})} = 1 + \frac{\tilde{\mathrm{V}}(\{\vec{a}\}) + \mathrm{I}(\{\vec{a}\})}{\mathrm{B}(\{\vec{a}\})} + \sum_{\{\widetilde{ij},\tilde{k}\}} \sum_{f_i=q,g} \int \mathrm{d}\Phi^{ij,k}_{R|B} \frac{\mathrm{R}_{ij,k}(r_{\widetilde{ij},\tilde{k}}(\{\vec{a}\})) - \mathrm{D}^{(\mathrm{S})}_{ij,k}(r_{\widetilde{ij},\tilde{k}}(\{\vec{a}\}))}{\mathrm{B}(\{\vec{a}\})}$$

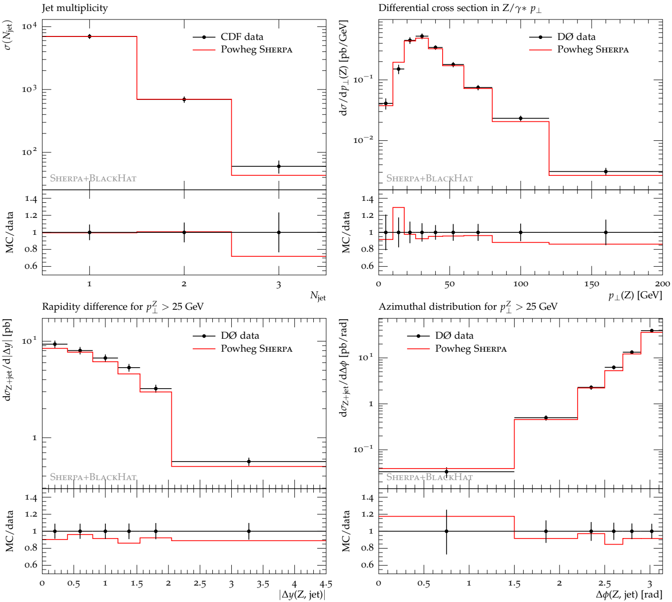
Note: Implies wrong dependence of observables on final-state momenta $\{\vec{p}\}$ $\rightarrow$ resolved by PS

Combine ME-correction and local K-factor $\Rightarrow$ observable O to $\mathcal{O}(\alpha_s)$

$$\langle O\rangle^{(\mathrm{POWHEG})} = \sum_{\{\vec{f}\}} \int \mathrm{d}\Phi_B(\{\vec{p}\})\,\bar{\mathrm{B}}(\{\vec{a}\}) \Bigg[\; \underbrace{\mathcal{P}(t_0;\{\vec{a}\})}_{\text{unresolved / virtual}} O(\{\vec{p}\})$$
$$+ \sum_{\{\widetilde{ij},\tilde{k}\}} \sum_{f_i=q,g} \frac{1}{16\pi^2} \int_{t_0} \mathrm{d}t \int_{z_{\min}}^{z_{\max}} \mathrm{d}z \int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi} J_{ij,k}(t,z,\phi)$$
$$\times \underbrace{\frac{1}{S_{ij}} \frac{S(r_{\widetilde{ij},\tilde{k}}(\{\vec{f}\}))}{S(\{\vec{f}\})} \frac{\mathrm{R}_{ij,k}(r_{\widetilde{ij},\tilde{k}}(\{\vec{a}\}))}{\mathrm{B}(\{\vec{a}\})} \mathcal{P}(t;\{\vec{a}\})}_{\text{resummed singular resolved}}\; O(r_{\widetilde{ij},\tilde{k}}(\{\vec{p}\}))\Bigg]$$

$\rightarrow$ **POWHEG method** [Nason] JHEP11(2004)040 [Frixione,Nason,Oleari] JHEP11(2007)070

POWHEG predictions from SHERPA



Large-N_C only

SHERPA POWHEG vs. Tevatron data

[CDF] PRL100(2008)102001

[DØ] PLB669(2008)278

[DØ] PLB682(2010)370

[Krauss,Schönherr,Siegert,SH] (will remain) unpublished

Fixed order vs. resummation: Part II - NLO matching

Exponentiate parts of R only → **MC@NLO** [Frixione,Webber] JHEP06(2002)029

$$\langle O\rangle^{(\mathrm{MC@NLO})} = \sum_{\{\vec f\}} \int \mathrm{d}\Phi_B(\{\vec p\})\, \bar{\mathrm{B}}^{(\mathrm{A})}(\{\vec a\}) \Bigg[\underbrace{\mathcal{P}^{(\mathrm{A})}(t_0;\{\vec a\})}_{\text{unresolved / virtual}} O(\{\vec p\})$$

$$+ \sum_{\{\widetilde{ij},\tilde k\}} \sum_{f_i=q,g} \frac{1}{16\pi^2} \int_{t_0} \mathrm{d}t \int_{z_{\min}}^{z_{\max}} \mathrm{d}z \int_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi}\, J_{ij,k}(t,z,\phi)$$

$$\times \underbrace{\frac{1}{S_{ij}} \frac{S(r_{\widetilde{ij},\tilde k}(\{\vec f\}))}{S(\{\vec f\})} \frac{\mathrm{D}^{(\mathrm{A})}_{ij,k}(r_{\widetilde{ij},\tilde k}(\{\vec a\}))}{\mathrm{B}(\{\vec a\})} \mathcal{P}^{(\mathrm{A})}(t;\{\vec a\})}_{\text{resummed singular resolved}} O(r_{\widetilde{ij},\tilde k}(\{\vec p\})) \Bigg]$$

$$+ \sum_{\{\vec f\}} \int \mathrm{d}\Phi_R(\{\vec p\}) \underbrace{\left[\mathrm{R}_{ij,k}(\{\vec a\}) - \mathrm{D}^{(\mathrm{A})}_{ij,k}(\{\vec a\}) \right]}_{\text{non-singular resolved}} O(\{\vec p\})$$

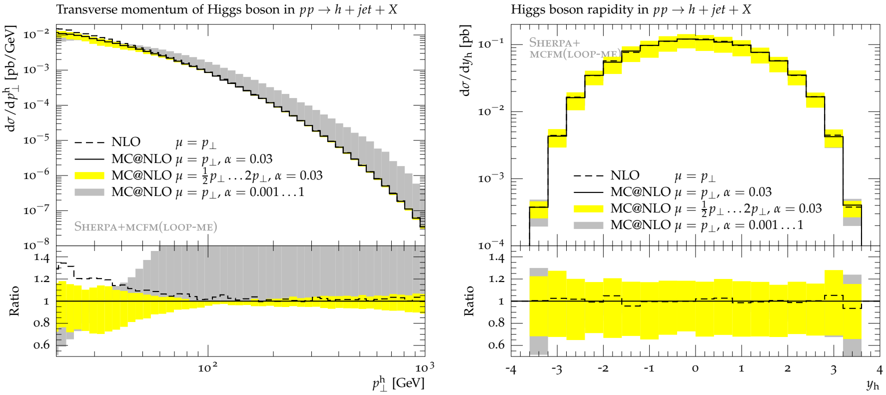
Seed cross sections change correspondingly as

$$\frac{\bar{\mathrm{B}}^{(\mathrm{A})}(\{\vec a\})}{\mathrm{B}(\{\vec a\})} = 1 + \frac{\tilde{\mathrm{V}}(\{\vec a\}) + \mathrm{I}(\{\vec a\})}{\mathrm{B}(\{\vec a\})} + \sum_{\{\widetilde{ij},\tilde k\}} \sum_{f_i=q,g} \int \mathrm{d}\Phi^{ij,k}_{R|B} \frac{\mathrm{D}^{(\mathrm{A})}_{ij,k}(r_{\widetilde{ij},\tilde k}(\{\vec a\})) - \mathrm{D}^{(\mathrm{S})}_{ij,k}(r_{\widetilde{ij},\tilde k}(\{\vec a\}))}{\mathrm{B}(\{\vec a\})}$$

Great simplification if $\mathrm{D}^{(\mathrm{A})}_{ij,k} \to \mathrm{D}^{(\mathrm{S})}_{ij,k}$ [Krauss,Schönherr,Siegert,SH] arXiv:1111.1220 [hep-ph]

Note: Must implement sub-leading colour terms in the parton shower!

MC@NLO predictions and matching uncertainties



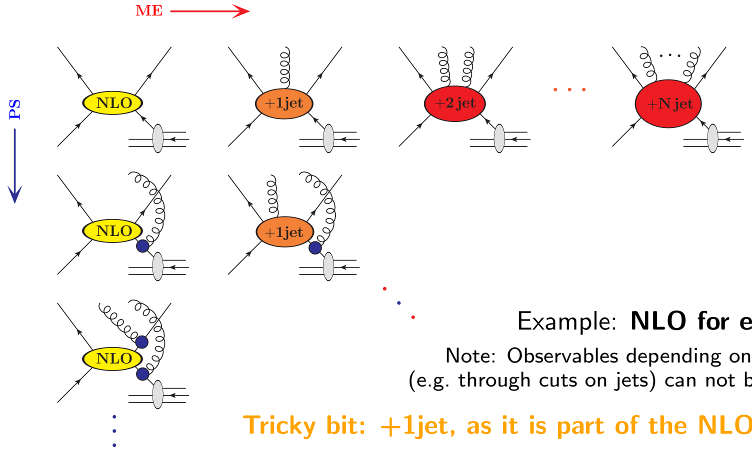
Example process: Higgs-boson plus jet production at the LHC
Assessment of matching uncertainties in POWHEG by varying $\mathrm{D}^{(\mathrm{A})}$

Upper edge of band ⇔ plain POWHEG, lower edge ⇔ MC@NLO
[Krauss,Schönherr,Siegert,SH] arXiv:1111.1220 [hep-ph]

To take home: Resumming small terms can make a big difference
Factorization scales must be respected in the parton shower

Fixed order vs. resummation: Part III - MENLOPS

Promoting the simulation to NLO accuracy for the core is currently cutting edge



Example: **NLO for $e^{\pm}q \to e^{\pm}q$**

Note: Observables depending on higher multi
(e.g. through cuts on jets) can not be improved !

Tricky bit: +1jet, as it is part of the NLO core, too

Fixed order vs. resummation: Part III - MENLOPS

Slice POWHEG/MC@NLO phase space in ME⊗PS-style

[Hamilton,Nason] JHEP06(2010)039, [Krauss,Schönherr,Siegert,SH] JHEP08(2011)123

$$
\begin{aligned}
\langle O\rangle^{(\mathrm{MENLOPS})} = \sum_{\{\vec f\}}\int \mathrm{d}\Phi_B(\{\vec p\})\,\bar{\mathrm{B}}(\{\vec a\})\Bigg[\,&\underbrace{\mathcal{P}(t_0;\{\vec a\})}_{\text{unresolved}}\;O(\{\vec p\})\\
+\sum_{\{\widetilde{ij},\tilde k\}}\sum_{f_i=q,g}\frac{1}{16\pi^2}\int_{t_0}\mathrm{d}t\int_{z_{\min}}^{z_{\max}}\mathrm{d}z\int_0^{2\pi}\frac{\mathrm{d}\phi}{2\pi}\,&J_{ij,k}(t,z,\phi)\;O(r_{\widetilde{ij},\tilde k}(\{\vec p\}))\\
\times\frac{1}{S_{ij}}\,\frac{S(r_{\widetilde{ij},\tilde k}(\{\vec f\}))}{S(\{\vec f\})}\,\frac{\mathrm{R}_{ij,k}(r_{\widetilde{ij},\tilde k}(\{\vec a\}))}{\mathrm{B}(\{\vec a\})}\Bigg(&\overbrace{\mathcal{P}(t;\{\vec a\})\,\Theta\left[Q_{\mathrm{cut}}-Q_{ij,k}(t,z,\phi)\right]}^{\text{resolved, PS domain}}\\
&+\underbrace{\mathcal{P}^{(\mathrm{PS})}(t;\{\vec a\})\,\Theta\left[Q_{ij,k}(t,z,\phi)-Q_{\mathrm{cut}}\right]}_{\text{resolved, ME domain}}\Bigg)\Bigg]
\end{aligned}
$$

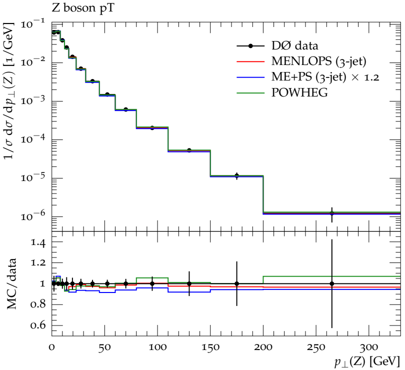
Note: Local K-factor $\bar{\mathrm{B}}/\mathrm{B}$ must be applied to ME⊗PS before merging

Overall characteristics

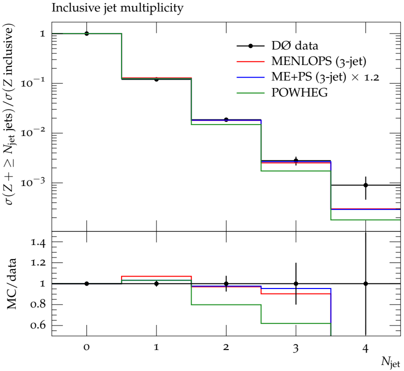
- Accuracy inherited from POWHEG/MC@NLO → stable rates for core process
- Higher-order tree-level via ME⊗PS → improved multi-jet predictions

MENLOPS for Z+jets

Z-boson p_T [DØ] PLB693(2010)522



Jet multiplicity [DØ] PLB658(2008)112



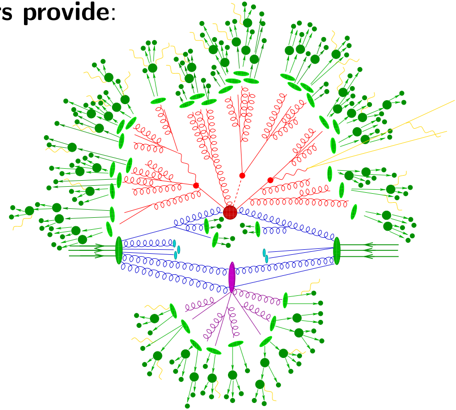
First Run II measurement using μ's
Data corrected to the particle level

SHERPA prediction: $p\bar{p} \to \ell\ell$@NLO
$\oplus$ $p\bar{p} \to \ell\ell+\{1,2,3\}$-jets@LO

Summary

Modern Monte-Carlo event generators provide:

- Reduced uncertainty due to fully/partially automated NLO MEs
- Largely reduced systematics with ME⊗PS, POWHEG & MC@NLO
- Reliable predictions for LEP, HERA, Tevatron and LHC



More and more higher-order pQCD built into MC event generators!

Models only where necessary mostly for non-perturbative aspects