Towards precision event simulation for collider experiments

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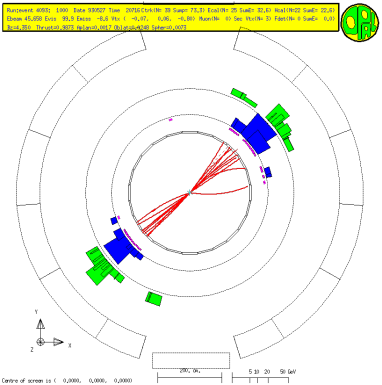
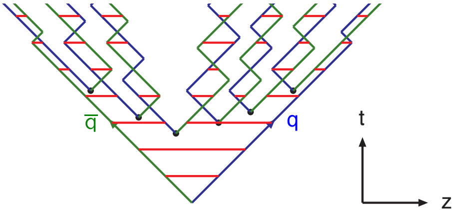
Event generators in 1979

[Andersson,Gustafson,Ingelman,Sjöstrand] Phys.Rept.97(1983)31





- Lund string model: $\sim$ like rubber band that is pulled apart and breaks into pieces, or like a magnet broken into smaller pieces.
- Complete description of 2-jet events in $e^+e^- \to$hadrons

Event generators in 1979

[Andersson,Gustafson,Ingelman,Sjöstrand] Phys.Rept.97(1983)31

```
      SUBROUTINE JETGEN(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /PAR/ PUD, PS1, SIGMA, CX2, EBEG, WFIN, IFLBEG
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      IFLSGN=(10-IFLBEG)/5
      W=2.*EBEG
      I=0
      IPD=0
C 1 FLAVOUR AND PT FOR FIRST QUARK
      IFL1=IABS(IFLBEG)
      PT1=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI1=6.2832*RANF(0)
      PX1=PT1*COS(PHI1)
      PY1=PT1*SIN(PHI1)
  100 I=I+1
C 2 FLAVOUR AND PT FOR NEXT ANTIQUARK
      IFL2=1+INT(RANF(0)/PUD)
      PT2=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI2=6.2832*RANF(0)
      PX2=PT2*COS(PHI2)
      PY2=PT2*SIN(PHI2)
C 3 MESON FORMED, SPIN ADDED AND FLAVOUR MIXED
      K(I,1)=MESO(3*(IFL1-1)+IFL2,IFLSGN)
      ISPIN=INT(PS1+RANF(0))
      K(I,2)=1+9*ISPIN+K(I,1)
      IF(K(I,1).LE.6) GOTO 110
      TMIX=RANF(0)
      KM=K(I,1)-6+3*ISPIN
      K(I,2)=8+9*ISPIN+INT(TMIX+CMIX(KM,1))+INT(TMIX+CMIX(KM,2))
C 4 MESON MASS FROM TABLE, PT FROM CONSTITUENTS
  110 P(I,5)=PMAS(K(I,2))
      P(I,1)=PX1+PX2
      P(I,2)=PY1+PY2
      PMTS=P(I,1)**2+P(I,2)**2+P(I,5)**2
C 5 RANDOM CHOICE OF X=(E+PZ)MESON/(E+PZ)AVAILABLE GIVES E AND PZ
      X=RANF(0)
      IF(RANF(0).LT.CX2) X=1.-X**(1./3.)
      P(I,3)=(X*W-PMTS/(X*W))/2.
      P(I,4)=(X*W+PMTS/(X*W))/2.
C 6 IF UNSTABLE, DECAY CHAIN INTO STABLE PARTICLES
  120 IPD=IPD+1
      IF(K(IPD,2).GE.8) CALL DECAY(IPD,I)
      IF(IPD.LT.I.AND.I.LE.96) GOTO 120
C 7 FLAVOUR AND PT OF QUARK FORMED IN PAIR WITH ANTIQUARK ABOVE
      IFL1=IFL2
      PX1=-PX2
      PY1=-PY2
C 8 IF ENOUGH E+PZ LEFT, GO TO 2
      W=(1.-X)*W
      IF(W.GT.WFIN.AND.I.LE.95) GOTO 100
      N=I
      RETURN
      END
```

```
      SUBROUTINE LIST(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /DATA3/ CHA1(9), CHA2(19), CHA3(2)
      WRITE(6,110)
      DO 100 I=1,N
      IF(K(I,1).GT.0) C1=CHA1(K(I,1))
      IF(K(I,1).LE.0) IC1=-K(I,1)
      C2=CHA2(K(I,2))
      C3=CHA3((47-K(I,2))/20)
      IF(K(I,1).GT.0) WRITE(6,120) I, C1, C2, C3, (P(I,J), J=1,5)
  100 IF(K(I,1).LE.0) WRITE(6,130) I, IC1, C2, C3, (P(I,J), J=1,5)
      RETURN
  110 FORMAT(////T11,'I',T17,'ORI',T24,'PART',T32,'STAB',
     &T44,'PX',T56,'PY',T68,'PZ',T80,'E',T92,'M'/)
  120 FORMAT(10X,I2,4X,A2,1X,2(4X,A4),5(4X,F8.1))
  130 FORMAT(10X,I2,4X,1X,I2,2(4X,A4),5(4X,F8.1))
      END
```

```
      SUBROUTINE DECAY(IPD,I)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      COMMON /DATA2/ IDCO(12), CBR(29), KDP(29,3)
      DIMENSION U(3), BE(3)
C 1 DECAY CHANNEL CHOICE, GIVES DECAY PRODUCTS
      TBR=RANF(0)
      IDC=IDCO(K(IPD,2)-7)
  100 IDC=IDC+1
      IF(TBR.GT.CBR(IDC)) GOTO 100
      ND=(59+KDP(IDC,3))/20
      DO 110 I1=I+1,I+ND
      K(I1,1)=-IPD
      K(I1,2)=KDP(IDC,I1-I)
  110 P(I1,5)=PMAS(K(I1,2))
C 2 IN THREE-PARTICLE DECAY CHOICE OF INVARIANT MASS OF PRODUCTS 2+3
      IF(ND.EQ.2) GOTO 130
      SA=(P(IPD,5)+P(I+1,5))**2
      SB=(P(IPD,5)-P(I+1,5))**2
      SC=(P(I+2,5)+P(I+3,5))**2
      SD=(P(I+2,5)-P(I+3,5))**2
      TDU=(SA-SD)*(SB-SC)/(4.*SQRT(SB*SC))
      IF(K(IPD,2).GE.11) TDU=SQRT(SB*SC)*TDU**3
  120 SX=SC+(SB-SC)*RANF(0)
      TDF=SQRT((SX-SA)*(SX-SB)*(SX-SC)*(SX-SD))/SX
      IF(K(IPD,2).GE.11) TDF=SX*TDF**3
      IF(RANF(0)*TDU.GT.TDF) GOTO 120
      P(100,5)=SQRT(SX)
C 3 TWO-PARTICLE DECAY IN CM, TWICE TO SIMULATE THREE-PARTICLE DECAY
  130 DO 160 IL=1,ND-1
      IO=(IL-1)*100-(IL-2)*IPD
      I1=I+IL
      I2=(ND-IL-1)*100-(ND-IL-2)*(I+IL+1)
      PA=SQRT((P(IO,5)**2-(P(I1,5)+P(I2,5))**2)*
     &(P(IO,5)**2-(P(I1,5)-P(I2,5))**2))/(2.*P(IO,5))
  140 U(3)=2.*RANF(0)-1.
      PHI=6.2832*RANF(0)
      U(1)=SQRT(1.-U(3)**2)*COS(PHI)
      U(2)=SQRT(1.-U(3)**2)*SIN(PHI)
      TDA=1.-(U(1)*P(IO,1)+U(2)*P(IO,2)+U(3)*P(IO,3))**2/
     &(P(IO,1)**2+P(IO,2)**2+P(IO,3)**2)
      IF(K(IPD,2).GE.11.AND.IL.EQ.2.AND.RANF(0).GT.TDA) GOTO 140
      DO 150 J=1,3
      P(I1,J)=PA*U(J)
  150 P(I2,J)=-PA*U(J)
      P(I1,4)=SQRT(PA**2+P(I1,5)**2)
  160 P(I2,4)=SQRT(PA**2+P(I2,5)**2)
C 4 DECAY PRODUCTS LORENTZ TRANSFORMED TO LAB SYSTEM
      DO 190 IL=ND-1,1,-1
      IO=(IL-1)*100-(IL-2)*IPD
      DO 170 J=1,3
  170 BE(J)=P(IO,J)/P(IO,4)
      GA=P(IO,4)/P(IO,5)
      DO 190 I1=I+IL,I+ND
      BEP=BE(1)*P(I1,1)+BE(2)*P(I1,2)+BE(3)*P(I1,3)
      DO 180 J=1,3
  180 P(I1,J)=P(I1,J)+GA*(GA/(1.+GA)*BEP+P(I1,4))*BE(J)
  190 P(I1,4)=GA*(P(I1,4)+BEP)
      I=I+ND
      RETURN
      END
```

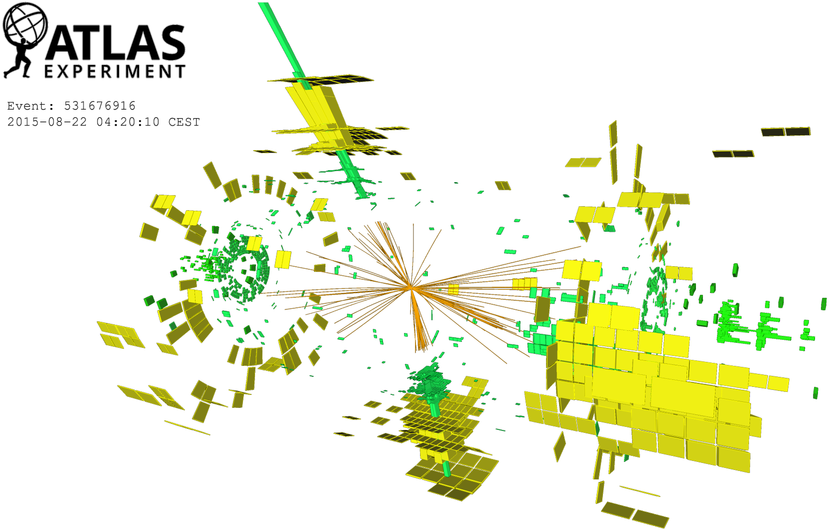
≈ 200 punched cards

Fortran code

```
      SUBROUTINE EDIT(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /EDPAR/ ITHROW, PZMIN, PMIN, THETA, PHI, BETA(3)
      REAL ROT(3,3), PR(3)
C 1 THROW AWAY NEUTRALS OR UNSTABLE OR WITH TOO LOW PZ OR P
      I1=0
      DO 110 I=1,N
      IF(ITHROW.GE.1.AND.K(I,2).GE.8) GOTO 110
      IF(ITHROW.GE.2.AND.K(I,2).GE.6) GOTO 110
      IF(ITHROW.GE.3.AND.K(I,2).EQ.1) GOTO 110
      IF(P(I,3).LT.PZMIN.OR.P(I,4)**2-P(I,5)**2.LT.PMIN**2) GOTO 110
      I1=I1+1
      K(I1,1)=IDIM(K(I,1),0)
      K(I1,2)=K(I,2)
      DO 100 J=1,5
  100 P(I1,J)=P(I,J)
  110 CONTINUE
      N=I1
C 2 ROTATE TO GIVE JET PRODUCED IN DIRECTION THETA, PHI
      IF(THETA.LT.1E-4) GOTO 140
      ROT(1,1)=COS(THETA)*COS(PHI)
      ROT(1,2)=-SIN(PHI)
      ROT(1,3)=SIN(THETA)*COS(PHI)
      ROT(2,1)=COS(THETA)*SIN(PHI)
      ROT(2,2)=COS(PHI)
      ROT(2,3)=SIN(THETA)*SIN(PHI)
      ROT(3,1)=-SIN(THETA)
      ROT(3,2)=0.
      ROT(3,3)=COS(THETA)
      DO 130 I=1,N
      DO 120 J=1,3
  120 PR(J)=P(I,J)
      DO 130 J=1,3
  130 P(I,J)=ROT(J,1)*PR(1)+ROT(J,2)*PR(2)+ROT(J,3)*PR(3)
C 3 OVERALL LORENTZ BOOST GIVEN BY BETA VECTOR
  140 IF(BETA(1)**2+BETA(2)**2+BETA(3)**2.LT.1E-8) RETURN
      GA=1./SQRT(1.-BETA(1)**2-BETA(2)**2-BETA(3)**2)
      DO 160 I=1,N
      BEP=BETA(1)*P(I,1)+BETA(2)*P(I,2)+BETA(3)*P(I,3)
      DO 150 J=1,3
  150 P(I,J)=P(I,J)+GA*(GA/(1.+GA)*BEP+P(I,4))*BETA(J)
  160 P(I,4)=GA*(P(I,4)+BEP)
      RETURN
      END
```

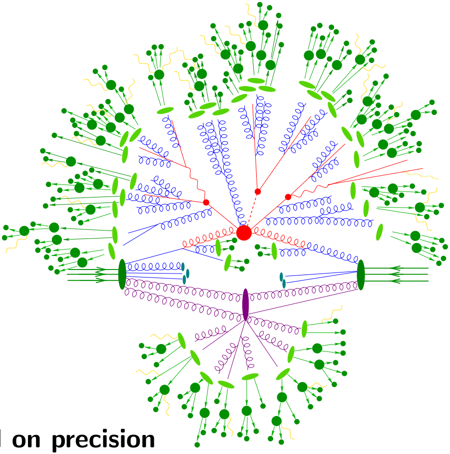
```
      BLOCK DATA
      COMMON /PAR/ PUD, PS1, SIGMA, CX2, EBEG, WFIN, IFLBEG
      COMMON /EDPAR/ ITHROW, PZMIN, PMIN, THETA, PHI, BETA(3)
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      COMMON /DATA2/ IDCO(12), CBR(29), KDP(29,3)
      COMMON /DATA3/ CHA1(9), CHA2(19), CHA3(2)
      DATA PUD/0.4/, PS1/0.5/, SIGMA/350./, CX2/0.77/,
     &EBEG/10000./, WFIN/100./, IFLBEG/1/
      DATA ITHROW/1/, PZMIN/0./, PMIN/0./, THETA,PHI,BETA/5*0./
      DATA MESO/7,1,3,2,8,5,4,6,9,7,2,4,1,8,6,3,5,9/
      DATA CMIX/2*0.5,1.,2*0.5,1.,2*0.25,0.5,2*0.,1./
      DATA PMAS/0.,2*139.6,2*493.7,2*497.7,135.,548.8,957.6,
     &2*765.9,2*892.2,2*896.3,770.2,782.6,1019.6/
      DATA IDCO/0,1,6,11,12,13,15,17,19,21,22,25/
      DATA CBR/1.,0.381,0.681,0.918,0.969,1.,0.426,0.662,0.959,
     &0.980,1.,1.,1.,0.667,1.,0.667,1.,0.667,1.,0.667,1.,1.,
     &0.899,0.987,1.,0.486,0.837,0.984,1./
      DATA KDP/1,1,8,2,1,1,2,8,1,1,1,2,3,6,4,7,5,4,6,5,7,2,2,
     &1,2,4,6,2,1,1,1,8,3,2,1,3,8,17,18,1,8,8,2,8,3,8,3,8,2,8,
     &3,3,8,3,5,7,3,9,0,0,8,8,3,8,9,9,14*0,8,4*0,8,0/
      DATA CHA1/'UD','DU','US','SU','DS','SD','UU','DD','SS'/
      DATA CHA2/'GAMM','PI+','PI-','K+','K-','K0','KB0','PI0','ETA',
     &'ETAP','RHO+','RHO-','K*+','K*-','K*0','KB*0','RHO0','OMEG','PHI'/
      DATA CHA3/'    ','STAB'/
      END
```

Experimental situation in 2019



Event generators in 2019

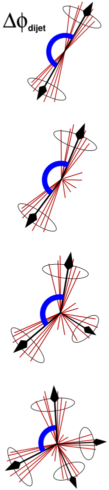
- LO Matrix Element generators and Loop-ME Generators
- Parton showers, mostly based on dipole/antenna picture
- Multiple interaction models possibly interleaved with shower
- Hadronization models string/cluster fragmentation
- Hadron decay packages
- Photon emission generators YFS formalism or QED shower

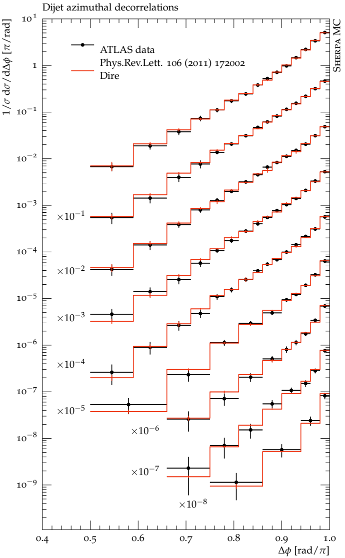


Much of the development focused on precision

Accuracy of the method: Jets at the LHC

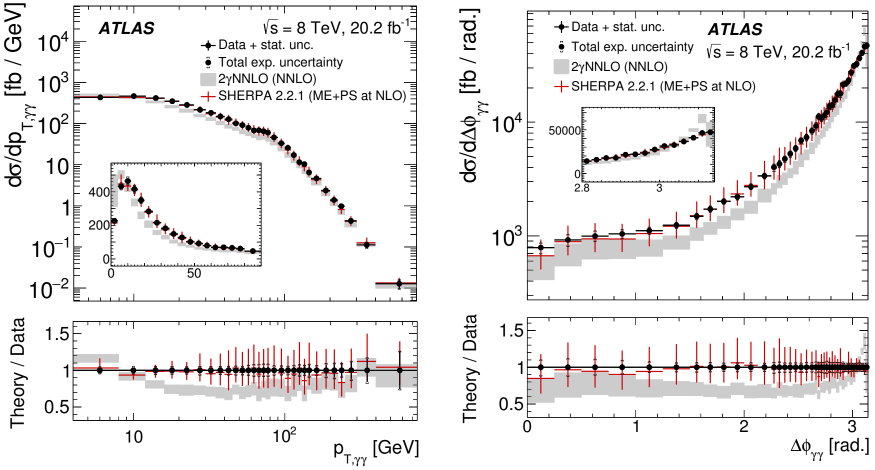
[Prestel,SH] arXiv:1506.05057





Accuracy of the method: Photons at the LHC

[ATLAS] arXiv:1704.03839



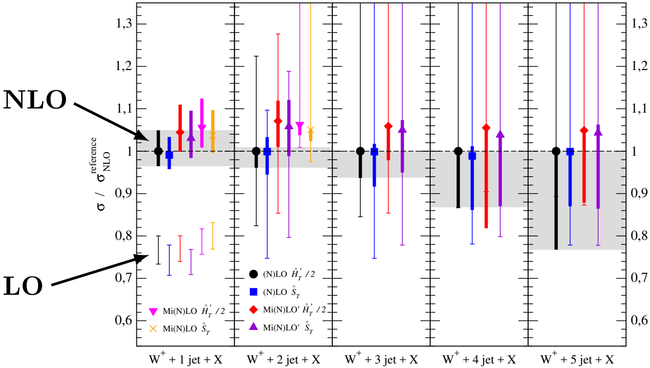
Accuracy versus precision

[A. David]

Fixed-order uncertainties

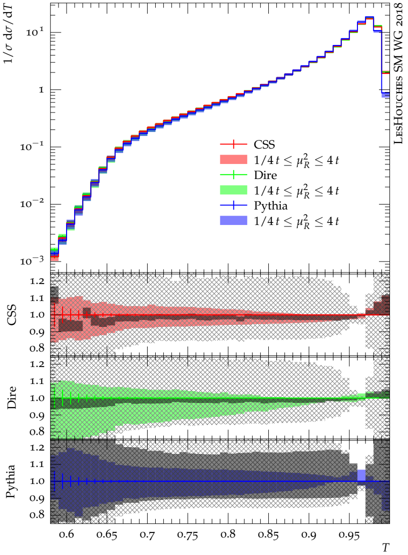
[Bern et al.] arXiv:1304.1253, arXiv:1412.4775
[Anger,Febres Cordero,Maître,SH] arXiv:1712.08621

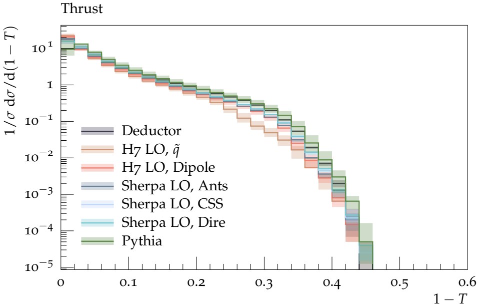
- $W^{\pm}$+jets at 13 TeV LHC, computed with BlackHat+Sherpa
- Largely reduced uncertainties at NLO, but more importantly good agreement for different functional forms of scale, including several variants of MINLO [Hamilton,Nason,Zanderighi] arXiv:1206.3572



Parton-shower uncertainties?

[LesHouches] arXiv:1605.04692, arXiv:1803.07977





- Assessment of PS uncertainties assuming they can be covered by varying evolution variable t in 2^{nd} order soft splitting kernel

$$\frac{1}{t}\left(\frac{\alpha_s(t)}{2\pi}\right)^2\left[\beta_0(t)\log\frac{k_T^2}{t}+K(t)\right]\frac{2}{1-z}$$

Outline of this Talk

Lessons learned

- Fueled by the NLO revolution, much of the MC community worked on precision fixed-order simulations during the last decade(s)
- This lead to much improved agreement with data and tremendous new capabilities of the generators, but left the resummation behind
- Many of the challenges at higher luminosity / energy require increased precision in the parton-shower simulation (we cannot hope to compute, say, 8-jet final states at NLO)

Towards a possible solution

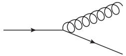
- Understand what precision means in the context of a parton shower
  - Parton shower is momentum conserving, analytics are not
  - Parton shower is unitary, analytic calculations mostly not
- Improve formal precision of parton shower, but keep its good features
  - Add higher-order corrections to splitting functions
  - Respect probability and momentum conservation

What is a parton shower?

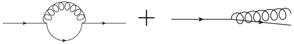
Radiative corrections as a branching process

[Marchesini,Webber] NPB238(1984)1, [Sjöstrand] PLB157(1985)321

- Make two well motivated assumptions
  - Parton branching can occur in two ways



\- observed



\- unobserved

  - Evolution conserves probability
- The consequence is Poisson statistics
  - Let the decay probability be λ
  - Assume indistinguishable particles $\rightarrow$ naive probability for n emissions

$$P_{\mathrm{naive}}(n,\lambda) = \frac{\lambda^n}{n!}$$

  - Probability conservation (i.e. unitarity) implies a no-emission probability

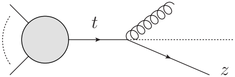
$$P(n,\lambda) = \frac{\lambda^n}{n!}\exp\{-\lambda\} \qquad \longrightarrow \qquad \sum_{n=0}^{\infty} P(n,\lambda) = 1$$

- In the context of parton showers $\Delta = \exp\{-\lambda\}$ is called Sudakov factor

Radiative corrections as a branching process

- Decay probability for parton state in collinear limit

$$\lambda \to \frac{1}{\sigma_n}\int_t^{Q^2} \mathrm{d}\bar{t}\,\frac{\mathrm{d}\sigma_{n+1}}{\mathrm{d}\bar{t}} \approx \sum_{\text{jets}}\int_t^{Q^2}\frac{\mathrm{d}\bar{t}}{\bar{t}}\int \mathrm{d}z\,\frac{\alpha_s}{2\pi}P(z)$$



Parameter t identified with evolution "time"

- Splitting function $P(z)$ spin & color dependent

$$P_{qq}(z) = C_F\left[\frac{2}{1-z} - (1+z)\right] \qquad P_{gq}(z) = T_R\left[\,z^2 + (1-z)^2\,\right]$$

$$P_{gg}(z) = C_A\left[\frac{2}{1-z} - 2 + z(1-z)\right] + (z \leftrightarrow 1-z)$$

- Matching to soft limit requires some care, because full soft emission probability present in all collinear sectors

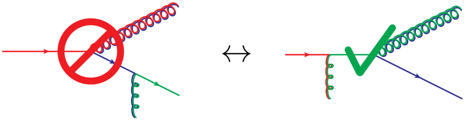
$$\frac{1}{t}\,\frac{2}{1-z} \xrightarrow{z\to 1} \frac{p_i p_k}{(p_i q)(q p_k)}$$

Soft double counting problem [Marchesini,Webber] NPB310(1988)461

Color coherence and the dipole picture

[Marchesini,Webber] NPB310(1988)461

- Individual color charges inside a color dipole cannot be resolved if gluon wavelength larger than dipole size → emission off "mother"

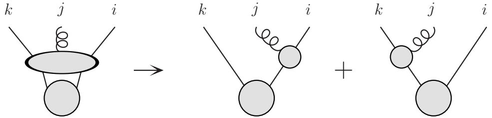


- Net effect is destructive interference outside cone with opening angle set by emitting color dipole → phase space for soft radiation halved

[Gustafsson,Pettersson] NPB306(1988)746

- Alternative description of effect in terms of dipole evolution
- Modern approach is to partial fraction soft eikonal and match to collinear sectors [Catani,Seymour] hep-ph/9605323

$$\frac{p_i p_k}{(p_i p_j)(p_j p_k)} \to \frac{1}{p_i p_j}\,\frac{p_i p_k}{(p_i+p_k)p_j} + \frac{1}{p_k p_j}\,\frac{p_i p_k}{(p_i+p_k)p_j}$$



Color coherence and the dipole picture

- Splitting kernels become dependent on anti-collinear direction
  usually defined by color spectator in large-N_c limit
- Singularity confined to soft-collinear region only
  captures all coherence effects at leading color, NLL

$$\frac{1}{1-z} \to \frac{1-z}{(1-z)^2+\kappa^2} \qquad \kappa^2 = \frac{k_\perp^2}{Q^2}$$

- Complete set of leading-order splitting functions now given by

$$P_{qq}(z,\kappa^2) = C_F\left[\frac{2\,(1-z)}{(1-z)^2+\kappa^2} - (1+z)\right]$$

$$P_{qg}(z,\kappa^2) = C_F\left[\frac{1+(1-z)^2}{z}\right], \qquad P_{gq}(z,\kappa^2) = T_R\left[z^2+(1-z)^2\right]$$

$$P_{gg}(z,\kappa^2) = 2\,C_A\left[\frac{1-z}{(1-z)^2+\kappa^2} + \frac{1}{z} - 2 + z(1-z)\right]$$

- Close correspondence to principal value regularization
  [Curci,Furmanski,Petronzio] NPB175(1980)27

What is “precision” in the parton-shower context?

What is “precision” in the parton-shower context?

- Parton shower proven to be NLL accurate for simple observables, provided that soft double-counting removed and 2-loop cusp anomalous dimension included [Catani,Marchesini,Webber] NPB349(1991)635
- More complicated observables require detailed analysis, leading color approximation and kinematics mapping typically problematic [Dasgupta,Dreyer,Hamilton,Monni,Salam] arXiv:1805.09327

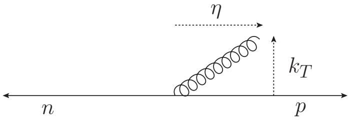
- For $>$25 years noone determined *numerically* the meaning of this
- Take a first stab at it:
  - Design a parton shower that reproduces NLL exactly
  - Figure out what differences are compared to plain PS
  - Assess effects one-by-one and compare numerically
- Diffcult due to vast amount of analytic results, hence
  - Keep it simple $\rightarrow$ additive observables in $e^+e^- \rightarrow$hadrons (i.e. Thrust, BKS [Berger,Kucs,Sterman], FC [Banfi,Salam,Zanderighi])
  - Use established, semi-analytic Caesar method as a reference [Banfi,Salam,Zanderighi] hep-ph/0407286

Kinematics and parametrization of observables

[Banfi,Salam,Zanderighi] hep-ph/0407286

- Contribution of one emission with momentum k to observable v

$$V(k) = \left(\frac{k_T}{Q}\right)^a e^{-b\eta} \qquad \leftarrow$$



where $k^\mu = (1-z)p^\mu + \beta n^\mu + k_T^\mu$ is soft-gluon momentum

- On-shell condition determines kinematics

$$\beta = \frac{k_T^2/Q^2}{1-z} \qquad \rightarrow \qquad \eta = \log\frac{1-z}{k_T/Q}$$

- Additive observables $\rightarrow$ emissions contribute as simple sum

$$v = V(\{p\},\{k\}) = \sum_i V(k_i)$$

- Prime example: Thrust in $e^+e^- \rightarrow$hadrons

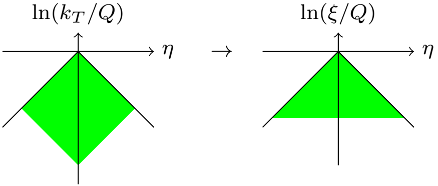
$$V(k) = \frac{p_-}{Q} = \frac{k_T}{Q}e^{-\eta} \qquad \rightarrow \qquad a=1, \quad b=1$$

NLL resummation for simple additive observables

[Banfi,Salam,Zanderighi] hep-ph/0407286

- Need one-emission probability for emissions harder than v to compute Sudakov factor $\to$ irregularly shaped region in k_T, η (Lund) plane
  Define suitable "evolution" variable to transform to a triangle

$$\xi = k_T^2 (1-z)^{-\frac{2b}{a+b}}$$



- One-emission probability becomes $\left(\xi = Q^2 v^{2/(a+b)}\right)$

$$R_{\rm NLL}(v) = 2\int_{Q^2 v^{\frac{2}{a+b}}}^{Q^2} \frac{d\xi}{\xi}\left[\int_0^1 dz\, \frac{\alpha_s\left(k_T^2\right)}{2\pi}\frac{2\,C_F}{1-z}\Theta(\eta) - \frac{\alpha_s(\xi)}{\pi}\,C_F B_q\right]$$

- Cumulative cross section $\Sigma(v) = 1/\sigma \int^v d\bar{v}(d\sigma/d\bar{v})$ given by

$$\Sigma_{\rm NLL}\left(v\right) = e^{-R_{\rm NLL}(v)}\mathcal{F}\left(v\right)$$

$\mathcal{F}(v) = \lim_{\epsilon\to 0}\mathcal{F}_\epsilon(v)$ is pure NLL, accounting for multiple emissions

Parton shower for simple additive observables

- Integrated one-emission probability in parton shower

$$R_{\mathrm{PS}}(v) = 2\int_{Q^2 v^{\frac{2}{a+b}}}^{Q^2} \frac{d\xi}{\xi} \int_{z_{\min}}^{z_{\max}} dz\, \frac{\alpha_s\left(k_T^2\right)}{2\pi}\, C_F \left[\frac{2}{1-z} - (1+z)\right] \Theta(\eta)$$

z-limits from momentum conservation, $\Theta(\eta)$ removes soft double-counting

- $\Sigma_{\mathrm{PS}}(v)$ determined by unitarity (i.e. Poisson statistics)
- Find unified NLL/PS expressions for $R(V)$ and $\Sigma(v)$

$$\Sigma(v) = \exp\left\{-\int_v \frac{d\xi}{\xi} R'_{>v}(\xi) - \int_{v_{\min}}^{v} \frac{d\xi}{\xi} R'_{<v}(\xi)\right\} \times \sum_{m=0}^{\infty} \frac{1}{m!} \left(\prod_{i=1}^{m} \int_{v_{\min}} \frac{d\xi_i}{\xi_i} R'_{<v}(\xi_i)\right) \Theta\left(v - \sum_{j=1}^{m} V(\xi_i)\right)$$

where

$$R'_{\lessgtr v}(\xi) = \frac{\alpha_s^{\lessgtr v,\mathrm{soft}}\left(\mu^2_{\lessgtr}\right)}{\pi} \int_{z^{\min}}^{z^{\max}_{\lessgtr v,\mathrm{soft}}} dz\, \frac{C_{\mathrm{F}}}{1-z} - \frac{\alpha_s^{\lessgtr v,\mathrm{coll}}\left(\mu^2_{\lessgtr v}\right)}{\pi} \int_{z^{\min}}^{z^{\max}_{\lessgtr v,\mathrm{coll}}} dz\, C_{\mathrm{F}}\, \frac{1+z}{2}$$

Differences between pure NLL and parton shower

[Reichelt,Siegert,SH] arXiv:1711.03497

- Isolate differences in terms of resolved/unresolved splitting probability:

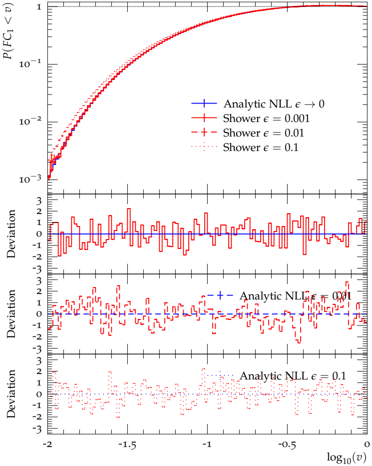
$$R'_{\lessgtr v}(\xi) = \frac{\alpha_s^{\lessgtr v,\mathrm{soft}}(\mu^2_{\lessgtr})}{\pi} \int_{z_{\min}}^{z^{\max}_{\lessgtr v,\mathrm{soft}}} dz\, \frac{C_\mathrm{F}}{1-z} - \frac{\alpha_s^{\lessgtr v,\mathrm{coll}}(\mu^2_{\lessgtr v})}{\pi} \int_{z_{\min}}^{z^{\max}_{\lessgtr v,\mathrm{coll}}} dz\, C_\mathrm{F}\, \frac{1+z}{2}$$

	NLL	Parton Shower
$z^{\max}_{>v,\mathrm{soft}}$	$1-(\xi/Q^2)^{\frac{a+b}{2a}}$	
$\mu^2_{>v,\mathrm{soft}}$	$\xi(1-z)^{\frac{2b}{a+b}}$	
$\alpha_s^{>v,\mathrm{soft}}$	2-loop CMW	
$z^{\max}_{<v,\mathrm{soft}}$	$1-v^{\frac{1}{a}}$	$1-(\xi/Q^2)^{\frac{a+b}{2a}}$
$\mu^2_{<v,\mathrm{soft}}$	$Q^2 v^{\frac{2}{a+b}}(1-z)^{\frac{2b}{a+b}}$	$\xi(1-z)^{\frac{2b}{a+b}}$
$\alpha_s^{<v,\mathrm{soft}}$	1-loop	2-loop CMW

	NLL	Parton Shower
$z^{\max}_{>v,\mathrm{coll}}$	1	$1-(\xi/Q^2)^{\frac{a+b}{2a}}$
$\mu^2_{>v,\mathrm{coll}}$	ξ	$\xi(1-z)^{\frac{2b}{a+b}}$
$\alpha_s^{>v,\mathrm{coll}}$	1-loop	2-loop CMW
$z^{\max}_{<v,\mathrm{coll}}$	0	$1-(\xi/Q^2)^{\frac{a+b}{2a}}$
$\mu^2_{<v,\mathrm{coll}}$	n.a.	$\xi(1-z)^{\frac{2b}{a+b}}$
$\alpha_s^{<v,\mathrm{coll}}$	n.a.	2-loop CMW

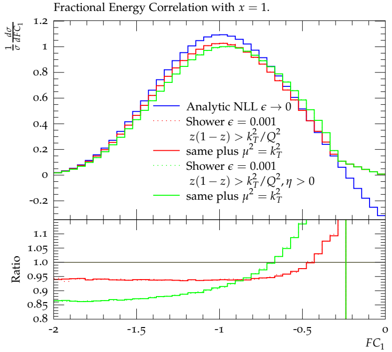
- Can cast pure NLL into PS language by using NLL expressions in PS
- Can study each effect in detail by reverting changes back to PS
- Dictionary for conversation between MC authors and theorists

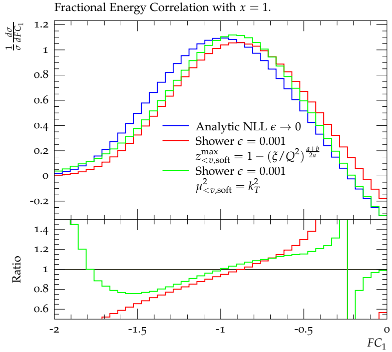
Baseline for comparison



- Modified parton shower exactly reproduces pure NLL result
- $E_{\rm cms}$=91.2 GeV, $\alpha_s(M_Z) = 0.118$ fixed flavor $n_f = 5$

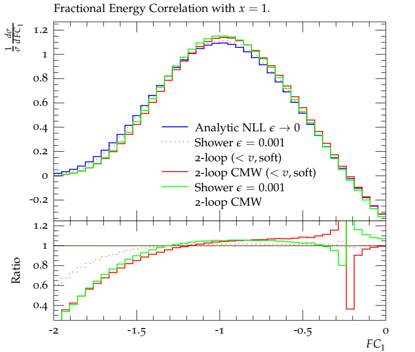
Local four momentum conservation and unitarity

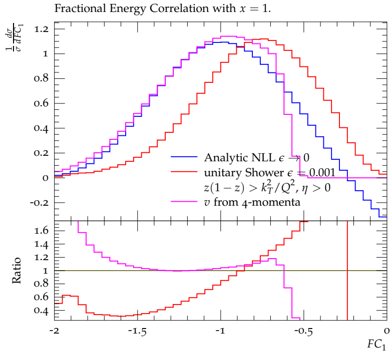




- NLL→PS in $z_{\min/\max}$
  (4-momentum conservation)
- NLL→PS in $z^{\max}_{>v,\text{coll}}$
  (phase-space sectorization)
- NLL→PS in $\mu^2_{>v,\text{coll}}$
  (conventional)
- NLL→PS in $z^{\max}_{<v,\text{soft}}$
  (from PS unitarity)
- NLL→PS in $\mu^2_{<v,\text{soft}}$
  (from PS unitarity)

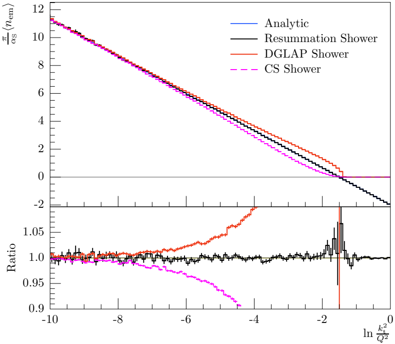
Running coupling and global momentum conservation

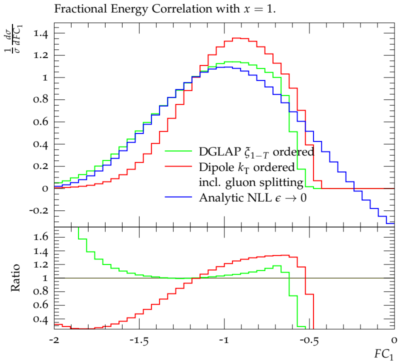




- NLL→PS in 2-loop CMW $< v, \text{soft}$ (from PS unitarity)
- NLL→PS in 2-loop CMW overall (conventional)
- NLL→PS in observable (use experimental definition)

Overall comparison NLL / PS / Dipole Shower





- Tuned comparison of differences between formally equivalent calculations
- Simplest process and simplest observable, but still sizable differences
- Origin of differences traced to treatment of kinematics & unitarity
- At NLL accuracy, none of the methods is formally better than another
  $\to$ Difference is a systematic uncertainty & needs to be kept in mind

How to make parton showers more precise?

Part I: Collinear limit

How to make parton showers more precise?

- Formulate parton-shower algorithm at NLO [Nagy,Soper] arXiv:1705.08093
  Naturally, NLO DGLAP evolution must be part of the full solution
- NLO DGLAP splitting kernels known since long
  [Curci,Furmanski,Petronzio] NPB175(1980)27, PLB97(1980)437
  [Floratos,Kounnas,Lacaze] NPB192(1981)417
- So far not implemented in parton showers because
  - NLO-calculation 4-2ε dimensional, but parton showers 4D
  - Overlap with soft-gluon resummation must be treated at NLO
- Focus on purely collinear corrections for a start
  Flavor-changing case is simplest but requires all the technology:
  - Redefine time-like Sudakovs to recover NLO DGLAP evolution
    [Jadach,Skrzypek] hep-ph/0312355
  - Phase-space factorization and kinematics for $2 \to 4$ transitions
    [Prestel,SH] arXiv:1705.00742
  - Negative NLO corrections $\to$ weighted veto algorithm
    [Schumann,Siegert,SH] arXiv:0912.3501, [Lönnblad] arXiv:1211.7204

Time-like parton showers and the DGLAP equation

- DGLAP equation for fragmentation functions

$$\frac{\mathrm{d}x\,D_a(x,t)}{\mathrm{d}\ln t} = \sum_{b=q,g}\int_0^1 \mathrm{d}\tau \int_0^1 \mathrm{d}z\,\frac{\alpha_s}{2\pi}\big[zP_{ab}(z)\big]_+\,\tau D_b(\tau,t)\,\delta(x-\tau z)$$

- Define plus prescription $\big[zP_{ab}(z)\big]_+ = \lim_{\varepsilon\to 0} zP_{ab}(z,\varepsilon)$

$$P_{ab}(z,\varepsilon) = P_{ab}(z)\,\Theta(1-z-\varepsilon) - \delta_{ab}\sum_{c\in\{q,g\}}\frac{\Theta(z-1+\varepsilon)}{\varepsilon}\int_0^{1-\varepsilon}\mathrm{d}\zeta\,\zeta\,P_{ac}(\zeta)$$

- Rewrite for finite ε

$$\frac{\mathrm{d}\ln D_a(x,t)}{\mathrm{d}\ln t} = -\sum_{c=q,g}\int\limits_0^{1-\varepsilon}\mathrm{d}\zeta\,\zeta\,\frac{\alpha_s}{2\pi}P_{ac}(\zeta) + \sum_{b=q,g}\int\limits_x^{1-\varepsilon}\frac{\mathrm{d}z}{z}\,\frac{\alpha_s}{2\pi}\,P_{ab}(z)\,\frac{D_b(x/z,t)}{D_a(x,t)}$$

- First term is logarithmic derivative of Sudakov factor

$$\Delta_a(t_0,t) = \exp\left\{-\int_{t_0}^{t}\frac{\mathrm{d}\bar{t}}{\bar{t}}\sum_{c=q,g}\int_0^{1-\varepsilon}\mathrm{d}\zeta\,\zeta\,\frac{\alpha_s}{2\pi}P_{ac}(\zeta)\right\}$$

Time-like parton showers and the DGLAP equation

- Use generating function $\mathcal{D}_a(x,t,\mu^2) = D_a(x,t)\Delta_a(t,\mu^2)$ to write

$$\frac{\mathrm{d}\ln\mathcal{D}_a(x,t,\mu^2)}{\mathrm{d}\ln t} = \sum_{b=q,g}\int_x^{1-\varepsilon}\frac{\mathrm{d}z}{z}\,\frac{\alpha_s}{2\pi}\,P_{ab}(z)\,\frac{D_b(x/z,t)}{D_a(x,t)}\;.$$

- A similar probability density is used to generate initial-state emissions
But final-state showers are typically unconstrained (hadrons not identified)
In this case the probability density is modified to

$$\frac{\mathrm{d}}{\mathrm{d}\ln t}\ln\left(\frac{\mathcal{D}_a(x,t,\mu^2)}{D_a(x,t)}\right) = \sum_{b=q,g}\int_0^{1-\varepsilon}\mathrm{d}z\,z\,\frac{\alpha_s}{2\pi}\,P_{ab}(z)\;.$$

- **Net result**: Unitarity implies that forward-branching Sudakovs must include a ‘symmetry factor’ z [Jadach,Skrzypek] hep-ph/0312355
- Convenient interpretation as “tagging” of evolving parton
- Equivalent to standard technique at LO due to symmetry of $P_{ab}(z)$
More care is needed at NLO [Prestel,SH] arXiv:1705.00742

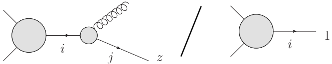
Collinear parton evolution at NLO

[Curci,Furmanski,Petronzio] NPB175(1980)27, [Floratos,Kounnas,Lacaze] NPB192(1981)417

- Higher-order DGLAP evolution kernels obtained from factorization

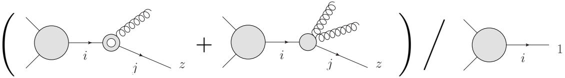
$$D^{(0)}_{ji}(z,\mu) = \delta_{ij}\delta(1-z) \qquad \leftrightarrow$$

$$D^{(1)}_{ji}(z,\mu) = -\frac{1}{\varepsilon}P^{(0)}_{ji}(z) \qquad \leftrightarrow$$



$$D^{(2)}_{ji}(z,\mu) = -\frac{1}{2\varepsilon}P^{(1)}_{ji}(z) + \frac{\beta_0}{4\varepsilon^2}P^{(0)}_{ji}(z) + \frac{1}{2\varepsilon^2}\int_z^1 \frac{\mathrm{d}x}{x} P^{(0)}_{jk}(x)P^{(0)}_{ki}(z/x)$$

$\leftrightarrow$

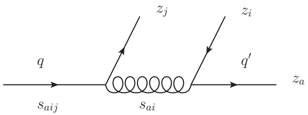


- $P^{(n)}_{ji}$ not probabilities, but sum rules hold ($\leftrightarrow$ unitarity constraint)
  In particular: Momentum sum rule identical between LO & NLO
- **Goal**: Perform the NLO computation of $P^{(1)}_{ji}$ fully differentially
  using modified dipole subtraction [Catani,Seymour] hep-ph/9605323

Collinear parton evolution at NLO

[Prestel,SH] arXiv:1705.00742

- Simulation of exclusive states requires computing splitting functions on the fly using differential NLO calculation & collinear factorization
- Schematically very similar to Catani-Seymour dipole subtraction
- Simplest example: Flavor-changing configuration $q \to q'$



Tree-level expression[1] $\leftrightarrow$ real-emission correction in CS

$$P_{qq'} = \frac{1}{2} C_F T_R \frac{s_{aij}}{s_{ai}} \left[-\frac{t_{ai,j}^2}{s_{ai} s_{aij}} + \frac{4\, z_j + (z_a - z_i)^2}{z_a + z_i} + (1 - 2\varepsilon) \left(z_a + z_i - \frac{s_{ai}}{s_{aij}} \right) \right]$$

Subtraction term (q→g)⊗(g→q') $\leftrightarrow$ differential subtraction term in CS

$$\tilde{P}_{qq'} = C_F T_R \frac{s_{aij}}{s_{ai}} \left(\frac{1 + \tilde{z}_j^2}{1 - \tilde{z}_j} - \varepsilon (1 - \tilde{z}_j) \right) \left(1 - \frac{2}{1 - \varepsilon} \frac{\tilde{z}_a \tilde{z}_i}{(\tilde{z}_a + \tilde{z}_i)^2} \right) + \ldots$$

[1] $(z_a + z_i)\, t_{ai,j} = 2(z_a s_{ij} - z_i s_{aj}) + (z_a - z_i) s_{ai}$

Collinear parton evolution at NLO

[Prestel,SH] arXiv:1705.00742

- Complete NLO result schematically given by

$$P^{(1)}_{qq'}(z) = \mathrm{C}_{qq'}(z) + \mathrm{I}_{qq'}(z) + \int \mathrm{d}\Phi_{+1}\Big[\mathrm{R}_{qq'}(z,\Phi_{+1}) - \mathrm{S}_{qq'}(z,\Phi_{+1})\Big]$$

- Real correction $R_{qq'}$ and subtraction terms $S_{qq'}$ $\nearrow$ previous slide
  Difference finite in 4 dimensions $\to$ amenable to MC simulation
- Integrated subtraction term and factorization counterterm given by

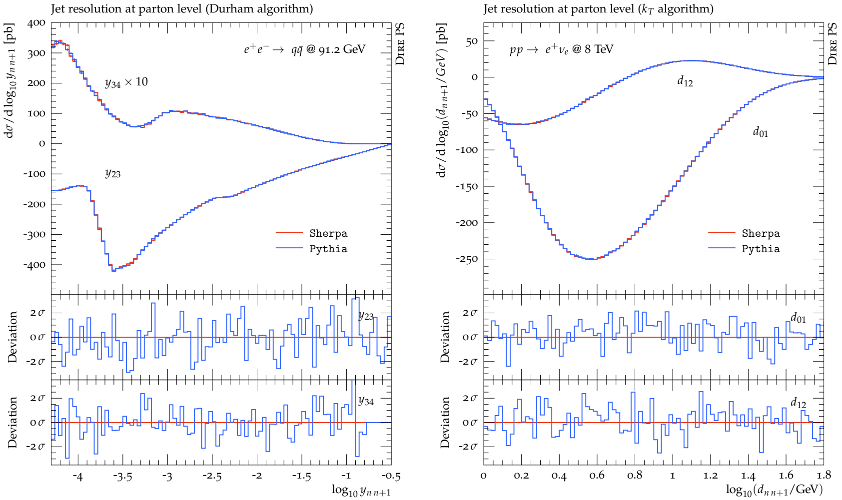
$$\mathrm{I}_{qq'}(z) = \int \mathrm{d}\Phi_{+1} S_{qq'}(z,\Phi_{+1})$$

$$\mathrm{C}_{qq'}(z) = \int_z \frac{\mathrm{d}x}{x}\left(P^{(0)}_{qg}(x) + \varepsilon\mathcal{J}^{(1)}_{qg}(x)\right)\frac{1}{\varepsilon}P^{(0)}_{gq}(z/x)$$

$$\mathcal{J}^{(1)}_{qg}(z) = 2C_F\left(\frac{1+(1-x)^2}{x}\ln(x(1-x)) + x\right)$$

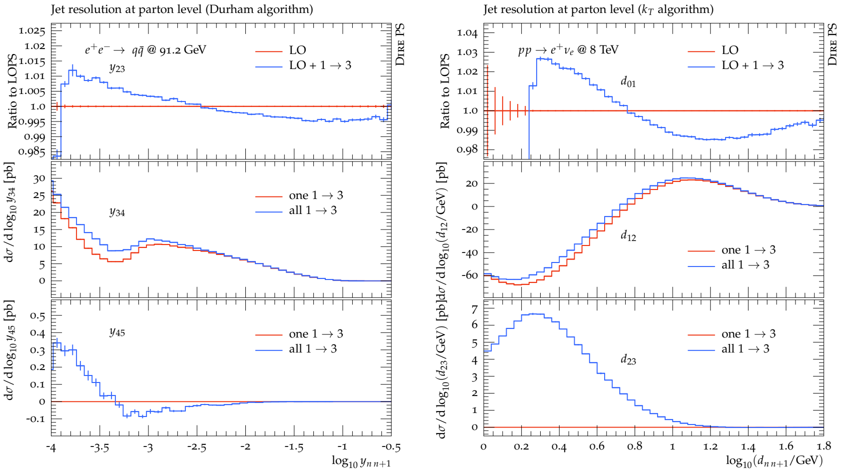
- Analytical computation of I not needed, as $\mathrm{I} + \mathcal{P}/\varepsilon$ finite
  generate as endpoint at $s_{ai} = 0$, starting from integrand at $\mathcal{O}(\varepsilon)$
- All components of $P^{(1)}_{qq'}$ eventually finite in 4 dimensions
  Can be simulated fully differentially in parton shower

Validation



- Effect of single $1 \to 3$ emission on leading and next-to-leading jet rate

Impact relative to leading-order prediction



- Effect of $1 \to 3$ emissions on leading jet rate
- Impact of multiple $1 \to 3$ emissions

How to make parton showers more precise?
Part II: Soft limit

Soft evolution at the next-to-leading order

[Marchesini,Korchemsky] PLB313(1993)433, hep-ph/9210281

- Soft-gluon resummed expression of Drell-Yan or DIS cross section

$$\frac{1}{\sigma}\frac{d\sigma(z,Q^2)}{d\log Q^2} = \mathcal{H}(Q^2)\,\widetilde{W}(z,Q^2)$$

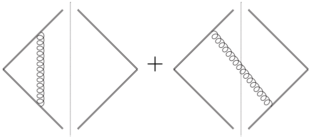
RGE governed by Wilson loop $\widetilde{W}$ ($Q(1-z)$ - total soft gluon energy)

- Non-abelian exponentiation theorem allows to expand as

$$\widetilde{W} = \exp\left\{\sum_{i=1}^{\infty} w^{(\mathrm{n})}\right\}$$

- One-loop result given by

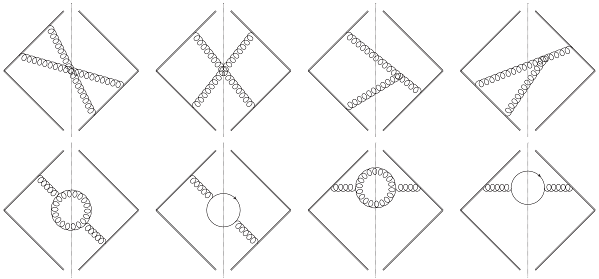
$$w^{(1)} = C_F\,\frac{\alpha_s(\mu)}{2\pi}\left[\ln^2 L + \frac{\pi^2}{6}\right] \qquad \leftrightarrow$$



where $L = -b_+ b_-/b_0^2$ and $b_0 = 2\,e^{-\gamma_E}/\mu$

Soft evolution at the next-to-leading order

- 2-loop contribution $w^{(2)}$ computed from (reals only) [Belitsky] hep-ph/9808389



- Renormalized result in position space

$$w^{(2)} = C_F \, \frac{\alpha_s^2(\mu)}{(2\pi)^2} \left[-\frac{\beta_0}{6} \ln^3 L + \Gamma^{(2)}_{\text{cusp}} \ln^2 L + 2 \ln L \left(\Gamma^{(2)}_{\text{soft}} + \frac{\pi^2}{12} \beta_0 \right) + \ldots \right]$$

$$\Gamma^{(2)}_{\text{cusp}} = \left(\frac{67}{18} - \frac{\pi^2}{6} \right) C_A - \frac{10}{9} \, T_R \, n_f \; , \qquad \beta_0 = \frac{11}{6} \, C_A - \frac{2}{3} \, T_R n_f$$

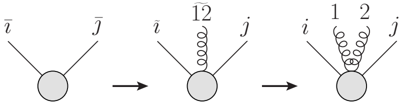
$$\Gamma^{(2)}_{\text{soft}} = \left(\frac{101}{27} - \frac{11}{72} \, \pi^2 - \frac{7}{2} \, \zeta_3 \right) C_A - \left(\frac{28}{27} - \frac{\pi^2}{18} \right) T_R \, n_f$$

- This is the benchmark to be reproduced by exclusive MC simulation

Separation of soft and collinear sectors

[Dulat,Prestel,SH] arXiv:1805.03757

- Phase space parametrized in terms of total soft momentum $q = p_1 + p_2$



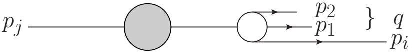
- Momentum space result expanded in Laurent series using

$$\frac{1}{q_\pm^{1+\epsilon}} = -\frac{1}{\epsilon}\,\delta(q_\pm) + \sum_{i=0}^{\infty} \frac{\epsilon^n}{n!} \left(\frac{\ln^n q_\pm}{q_\pm}\right)_+$$

- Unitarity implies that factorized plus distributions like $[1/q_+]_+[1/q_-]_+$ have no PS analogue $\rightarrow$ define double-plus distributions instead

$$\big[f(q_+,q_-)\big]_{++}\, g(q_+,q_-) = f(q_+,q_-)\Big(g(q_+,q_-) - g(0,0)\Big)$$

- Re-organize entire calculation in terms of pure soft & collinear terms
  Key observation: $q_\pm = 0$ implies collinear limit for 1 & 2 emissions



Soft evolution at the next-to-leading order

[Catani,Grazzini] hep-ph/9908523

- Real-emission corrections can be written in convenient form

$$\mathcal{S}_{ij}^{(q\bar{q})}(1,2) = -\frac{s_{ij}}{(s_{i1}+s_{i2})(s_{j1}+s_{j2})}\frac{T_R}{s_{12}}\Big(1-4\,z_1 z_2\cos^2\phi_{12,ij}\Big)$$

$$\mathcal{S}_{ij}^{(gg)}(1,2) = \mathcal{S}_{ij}^{(\mathrm{s.o.})}(1,2)\,\frac{C_A}{2}\left(1+\frac{s_{i1}s_{j1}+s_{i2}s_{j2}}{(s_{i1}+s_{i2})(s_{j1}+s_{j2})}\right) + \frac{s_{ij}}{(s_{i1}+s_{i2})(s_{j1}+s_{j2})}\frac{C_A}{s_{12}}\Big(-2+4\,(1-\epsilon)\,z_1 z_2\cos^2\phi_{12,ij}\Big)$$

- Strongly ordered and spin correlation components

$$\mathcal{S}_{ij}^{(\mathrm{s.o.})}(1,2) = \frac{s_{ij}}{s_{i1}s_{12}s_{j2}} + \frac{s_{ij}}{s_{j1}s_{12}s_{i2}} - \frac{s_{ij}^2}{s_{i1}s_{j1}s_{i2}s_{j2}}$$

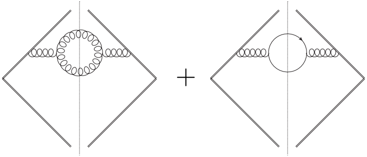
$$4\,z_1 z_2\cos^2\phi_{12,ij} = \frac{(s_{i1}s_{j2}-s_{i2}s_{j1})^2}{s_{12}s_{ij}(s_{i1}+s_{i2})(s_{j1}+s_{j2})}$$

- Apparently simple structure, but unlike collinear NLO results not reflected by iterated leading-order splitting kernels
  $\rightarrow$ not all denominators can be composed from LO expressions

NLO subtraction: Dipole approach

[Dulat,Prestel,SH] arXiv:1805.03757

- Nearly ok subtraction obtained from spin correlated parton shower



$$= \sum_{b=q,g} J_{ij,\mu}(p_{12}) J_{ij,\nu}(p_{12}) \frac{P^{\mu\nu}_{gb}(z_1)}{s_{12}}$$

- Building blocks are eikonal currents

$$J^{\mu}_{ij}(q) = \frac{p_i^\mu}{2p_i q} - \frac{p_j^\mu}{2p_j q}$$

and collinear splitting functions

$$P^{\mu\nu}_{gq}(z) = T_R \left(-g^{\mu\nu} + 4\, z(1-z)\, \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} \right)$$

$$P^{\mu\nu}_{gg}(z) = C_A \left(-g^{\mu\nu} \left(\frac{z}{1-z} + \frac{1-z}{z} \right) - 2\,(1-\varepsilon) z(1-z)\, \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} \right)$$

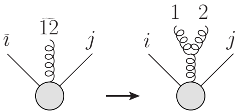
- Finite remainder has integrable singularities $\rightarrow$ not suitable for MC problem arises from interference of abelian & non-abelian diagrams

NLO subtraction: Antenna approach – Kinematics

[Dulat,Prestel,SH] arXiv:1805.03757

- In iterated emission $\tilde{\imath}\,\tilde{\jmath} \to \tilde{\imath}\,\widetilde{12}\,j \to ij12$ emission probability of first step written in terms of momenta after second step is

$$\frac{\tilde{p}_i p_j}{2(\tilde{p}_i \tilde{p}_{12})(\tilde{p}_{12} p_j)} = \frac{s_{ij}}{(s_{i1}+s_{i2})(s_{j1}+s_{j2}) - s_{ij}s_{12}}$$



- Not identical to desired "eikonal" $s_{ij}/((s_{i1}+s_{i2})(s_{j1}+s_{j2}))$ in soft⊗collinear terms of $\mathcal{S}_{ij}$ but easily corrected by weight

$$w_{ij}^{12} = 1 - \frac{s_{ij}s_{12}}{(s_{i1}+s_{i2})(s_{j1}+s_{j2})} = \left(\frac{p_{\perp,12}^{(ij)}}{m_{\perp,12}^{(ij)}}\right)^2$$

- Iterated eikonals of type $s_{ij}/(s_{i1}s_{j1})$, $s_{j1}/(s_{12}s_{j2})$ in $\mathcal{S}_{ij}^{(\mathrm{s.o.})}$ reconstructed by partial fractioning & matching to LO^2 $\to$ additional weight

$$\bar{w}_{ij}^{12} = \frac{(s_{i1}+s_{i2})(s_{j1}+s_{j2}) - s_{ij}s_{12}}{s_{i1}s_{j1}+s_{i2}s_{j2}} = \frac{\left(p_{\perp,12}^{(ij)}\right)^2}{\left(p_{\perp,1}^{(ij)}\right)^2 + \left(p_{\perp,2}^{(ij)}\right)^2}$$

- These weights lie between zero and one and reduce emission rates

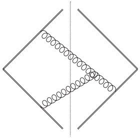
Leading color fully differential soft evolution at NLO

[Dulat,Prestel,SH] arXiv:1805.03757

- Squared LO eikonal and negative term in $\mathcal{S}_{ij}^{(\mathrm{s.o.})}$ both have no parton-shower analogue $\to$ correct for both mismatches by adding sub-leading color contribution to $i1$-collinear splitting functions

$$P_{ij,A}^{(\mathrm{slc})}(1,2) = \frac{2\,s_{ij}}{s_{i1}+s_{j1}}\,\frac{w_{ij}^{12}+\bar{w}_{ij}^{12}}{2}\Big(\bar{C}_{ij}-C_A\Big)\;,\quad \bar{C}_{ij} = \begin{cases} 2C_F & \text{if } i \;\&\; j \text{ quarks} \\ C_A & \text{else} \end{cases}$$

- Second soft emission off Wilson lines occurs with color charge factor C_A due to interference with octet



$$P_{ij,B}^{(\mathrm{slc})}(1,2) = \frac{2\,s_{i2}}{s_{i1}+s_{12}}\,\frac{w_{ij}^{12}+\bar{w}_{ij}^{12}}{2}\,\Big(C_A-\bar{C}_{ij}\Big)$$

- Combined effect on $i1$-collinear matched splitting function

$$P_{ij}^{(\mathrm{slc})}(1,2) = \Big(C_A-\bar{C}_{ij}\Big)\left(\frac{2\,s_{i2}}{s_{i1}+s_{12}}-\frac{2\,s_{ij}}{s_{i1}+s_{j1}}\right)\frac{w_{ij}^{12}+\bar{w}_{ij}^{12}}{2}$$

- Non-singular in $i1$-collinear limit $\to$ color charges of Wilson lines in soft-collinear limit are C_i and C_j, in agreement with DGLAP

Leading color fully differential soft evolution at NLO

[Dulat,Prestel,SH] arXiv:1805.03757

- Complete NLO-weighted LO splitting functions

$$(P_{qq})_i^k(1,2) = C_F\left(\frac{2\,s_{i2}}{s_{i1}+s_{12}}\,\frac{w_{ik}^{12}+\bar{w}_{ik}^{12}}{2}\right) + P_{ik}^{(\mathrm{slc})}(1,2)$$

$$(P_{gg})_{ij}(1,2) = C_A\left(\frac{2\,s_{i2}}{s_{i1}+s_{12}}\,\frac{w_{ij}^{12}+\bar{w}_{ij}^{12}}{2} + w_{ij}^{12}\left(-1+z(1-z)\,2\cos^2\phi_{12}^{ij}\right)\right)$$

$$(P_{gq})_{ij}(1,2) = T_R\,w_{ij}^{12}\left(1-4z(1-z)\cos^2\phi_{12}^{ij}\right)$$

- Calculation completed by subtracted real correction, virtuals and factorization counterterms
- Counterterms are endpoint contributions, as in collinear limit

$$\tilde{\mathcal{S}}_{gq}^{(\mathrm{cusp})} = \delta(s_{12})\,\frac{2\,s_{ij}}{s_{i12}s_{j12}}\,T_R\Big[2z(1-z)+\big(1-2z(1-z)\big)\ln(z(1-z))\Big]$$

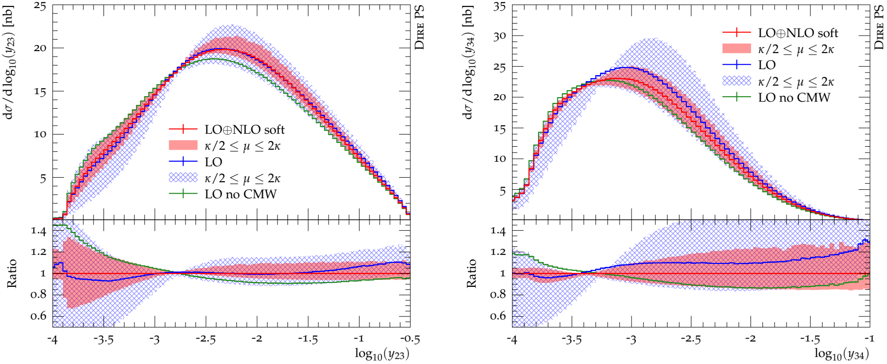
$$\tilde{\mathcal{S}}_{gg}^{(\mathrm{cusp})} = \delta(s_{12})\,\frac{2\,s_{ij}}{s_{i12}s_{j12}}\,2C_A\left[\frac{\ln z}{1-z}+\frac{\ln(1-z)}{z}+\big(-2+z(1-z)\big)\ln(z(1-z))\right]$$

$$\tilde{\mathcal{S}}_{wl}^{(\mathrm{cusp})} = -\,\delta(s_{i1})\,\frac{1}{2}\,\frac{C_A}{2}\,\frac{2\,s_{ij}}{s_{i12}s_{j12}}\left(\frac{\ln z_i}{1-z_i}+\frac{\ln(1-z_i)}{z_i}\right)+\big(\text{swaps}\big)$$

Sum integrates to CMW correction [Catani,Marchesini,Webber] NPB349(1991)635

Leading color fully differential soft evolution at NLO

[Dulat,Prestel,SH] arXiv:1805.03757



- Impact on $2 \to 3$ and $3 \to 4$ Durham jet rate at LEP I
- Uncertainty bands no longer just estimates
  but perturbative QCD predictions for the first time
- Fair agreement with CMW scheme

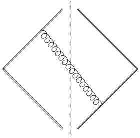
How to make parton showers more precise?
Part III: Beyond leading color

Parton-shower formalism at full color

[Reichelt,SH] arXiv:19mm.soon

- Evolution of matrix element in strongly ordered soft limit determined by

$$\mathbf{\Gamma}_n(\mathbf{\Gamma}) = -\sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n} \mathbf{T}_i\,\mathbf{\Gamma}\,\mathbf{T}_j\,w_{ij}\;, \qquad w_{ij} = \frac{s_{ij}}{s_{iq}s_{jq}} \quad \leftrightarrow$$



- Multiple soft insertions lead to $k+1$-gluon matrix element

$$\langle m_{n+k+1}|m_{n+k+1}\rangle = \langle M_n|\mathbf{\Gamma}_n(\mathbf{\Gamma}_{n+1}(\ldots\mathbf{\Gamma}_{n+k+1}(\mathbf{\Gamma}_{n+k}(\mathbf{1}))\ldots))|M_n\rangle$$

- Differential radiation probability becomes

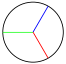
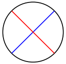
$$\frac{\mathrm{d}\sigma_{n+k+1}}{\sigma_{n+k}} = \mathrm{d}\Phi_{+1}\,8\pi\alpha_s\,\frac{\langle m_{n+k}|\mathbf{\Gamma}_{n+k}(\mathbf{1})|m_{n+k}\rangle}{\langle m_{n+k}|m_{n+k}\rangle}\;,$$

- In general, this differs from the terms leading to the soft function in approaches like Caesar [Banfi,Salam,Zanderighi] hep-ph/0407286, but is identical for the global recursively IR safe observables considered there

Computation of color insertions

[Reichelt,SH] arXiv:19mm.soon

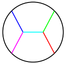
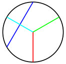
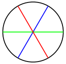
- An essential ingredient for the MC implementation is an algorithm to compute $\mathbf{\Gamma}$ that scales linearly with the number of gluons
- This can be achieved by working in the bi-fundamental representation and sampling over color assignments $\leftrightarrow$ color flow representation

Coefficient	Analytic value / N_c	MC result / N_c
 $= F^c_{ab}\mathrm{Tr}\left[T^aT^bT^c\right]$	$C_F\dfrac{C_A}{2}$	1.9998(2)
 $= \mathrm{Tr}\left[T^aT^bT^aT^b\right]$	$-C_F\left(\dfrac{C_A}{2}-C_F\right)$	-0.2221(1)

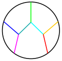
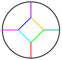
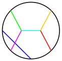
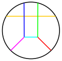
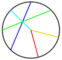
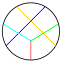
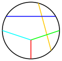
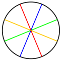
Computation of color insertions

[Reichelt,SH] arXiv:19mm.soon

- An essential ingredient for the MC implementation is an algorithm to compute $\mathbf{\Gamma}$ that scales linearly with the number of gluons
- This can be achieved by working in the bi-fundamental representation and sampling over color assignments $\leftrightarrow$ color flow representation

Coefficient	Analytic value / N_c	MC result / N_c
 $= F^d_{ae} F^c_{eb} \mathrm{Tr}\left[T^a T^b T^c T^d\right]$	$C_F \left(\frac{C_A}{2}\right)^2$	2.9995(4)
 $= F^d_{bc} \mathrm{Tr}\left[T^a T^b T^a T^c T^d\right]$	$-C_F \frac{C_A}{2}\left(\frac{C_A}{2} - C_F\right)$	-0.3332(3)
 $= \mathrm{Tr}\left[T^a T^b T^c T^a T^b T^c\right]$	$C_F \left(\frac{C_A}{2} - C_F\right)(C_A - C_F)$	0.3701(1)

Computation of color insertions

Coefficient	Analytic value / N_c	MC result / N_c
	$C_F \left(\frac{C_A}{2}\right)^3$	4.4996(8)
	$C_F \left(\frac{C_A}{2}\right)^3 \left(1 + \frac{2}{N_c^2}\right)$	5.499(1)
	$-C_F \left(\frac{C_A}{2}\right)^2 \left(\frac{C_A}{2} - C_F\right)$	-0.5001(5)
	$-C_F \left(\frac{C_A}{2}\right)^2 \left(\frac{C_A}{2} - C_F - \frac{C_A}{N_c^2}\right)$	0.5007(4)
	$C_F \frac{C_A}{2} \left(\frac{C_A}{2} - C_F\right)(C_A - C_F)$	0.5556(2)
	$C_F \frac{C_A}{2} \left(\left(\frac{C_A}{2} - C_F\right)(C_A - C_F) - \frac{C_A^2}{2N_c^2}\right)$	-0.4446(2)
	$C_F \frac{C_A}{2} \left(\frac{C_A}{2} - C_F\right)^2$	0.0558(3)
	$-C_F \left(\left(\frac{C_A}{2} - C_F\right)(C_A - C_F)\left(\frac{3}{2}C_A - C_F\right) - \frac{C_A^3}{4N_c^2}\right)$	-0.1729(1)

Rearrangement of antenna functions

[Reichelt,SH] arXiv:19mm.soon

- Another essential ingredient for the MC implementation is an efficient organization of the splitting functions
- Can borrow from the computation of the double-soft function
  [Dulat,Prestel,SH] arXiv:1805.03757

$$\mathbf{\Gamma}_n(\mathbf{\Gamma}) = \frac{1}{n-1}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\mathbf{T}_i\,\mathbf{\Gamma}\,\mathbf{T}_i P^i_j + \frac{1}{2}\sum_{\substack{k=1\\ k\neq i,j}}^{n}\Big(\mathbf{T}_i\,\mathbf{\Gamma}\,\mathbf{T}_k + \mathbf{T}_k\,\mathbf{\Gamma}\,\mathbf{T}_i\Big)\tilde{P}^i_{jk}\right)$$

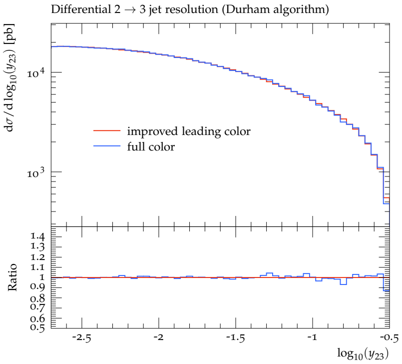
  where

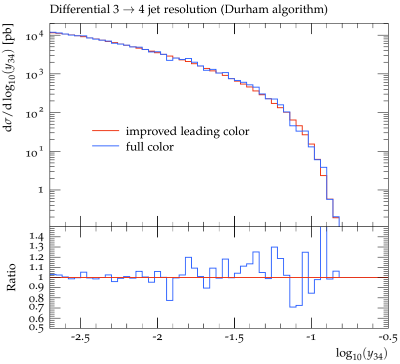
$$P^i_j = \frac{1}{s_{iq}}\frac{2\,s_{ij}}{s_{iq}+s_{jq}}\;, \qquad \tilde{P}^i_{jk} = P^i_j - P^i_k$$

- $\tilde{P}^i_{jk}$ tends to zero in the iq-collinear limit
  $\rightarrow$ sub-leading color corrections kinematically suppressed

Towards full color evolution

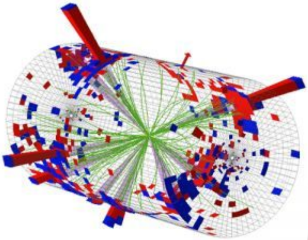
- Example: Durham k_T-jet rates at LEP (91.2 GeV)
- These plots were generated with 32h worth of CPU time

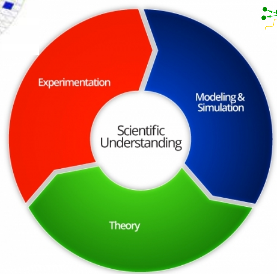


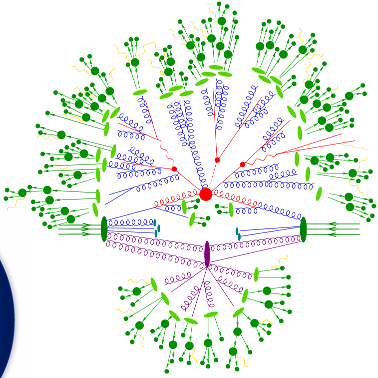


Towards precision event simulation

Event generators in the bigger picture





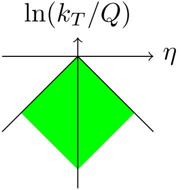


$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \not{D} \psi + h.c.$$

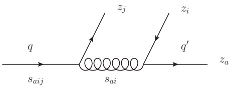
Some history of parton-showers (personal view)

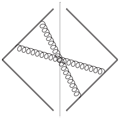
Assembling the pieces into precision tools

- Relation to analytic resummation understood agreement can be restored by eliminating momentum & probability conservation in PS

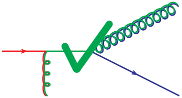


- NLO corrections to collinear and soft evolution to be included in fully exclusive simulation





- Angular ordering approximation to coherence to be replaced by color dipole picture combined with full-color evolution



Summary & Outlook

Parton shower algorithms

- Crucial ingredient to full event simulation
- Essential to getting jet shapes right, thus feeding into event shapes, acceptances, subjet multiplicity & structure
- Many technical and formal improvements over past years renewed interest in dipole formulation and improvements thereof
- Parton showers undergoing similar transformation right now as fixed-order calculations two decades ago $\rightarrow$ 2^{nd} NLO revolution

Parton shower resummation

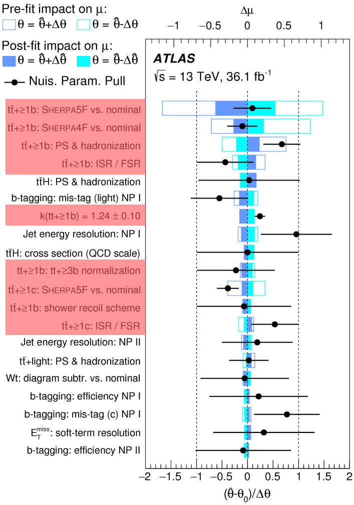
- (Dis-)agreement with analytic approaches worked out in simple cases
- Vibrant discussion of formal accuracy of parton showers spurred by experimental needs and theoretical interest
- Many open questions, but much progress recently

Thank you for your attention

Status of $t\bar{t}b\bar{b}$

[F.Siegert SM@LHC '19]

- ATLAS and CMS $t\bar{t}H(b\bar{b})$ analyses rely on MC modelling for irreducible $t\bar{t}b\bar{b}$ BG
- Largest sources of uncertainty on extracted signal strength related to $t\bar{t}$+HF modeling!
- What can be improved?
  - ATLAS & CMS: relied on NLO+PS $t\bar{t}$ so far! More accurate theory with NLO $t\bar{t}b\bar{b}$ used only to reweight HF fractions (ATLAS) or cross-checks (CMS)
  - Theory: Large perturbative $t\bar{t}b\bar{b}$ uncertainties even increased by NLO+PS algorithms
  - Both: More rigorous combination of inclusive $t\bar{t}$+jets and $t\bar{t}b\bar{b}$ predictions.



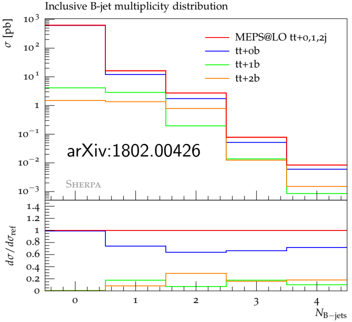
Status of $t\bar{t}b\bar{b}$

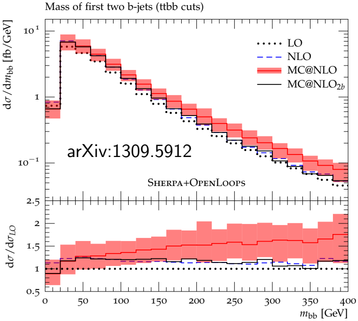
Traditional MC simulation approaches to $t\bar{t}$+HF

- **Five-flavor scheme**:
  - "Inclusive" NLO+PS $t\bar{t}$ sample with HF from parton shower $g \to b\bar{b}$
  - Multi-leg merged $t\bar{t}$+jets sample with HF from higher-order MEs (hard b) or parton shower $g \to b\bar{b}$ (soft/coll b)

  Surprising feature:
  - Jet production described by hard MEs, but b-jets not always from b-MEs!
  - soft/collinear $g \to b\bar{b}$ from PS can transform light jets into b-jets
- **Four-flavor scheme**:
  - NLO+PS $t\bar{t}b\bar{b}$ using matrix elements with massive b-quarks

  Surprising feature:
  - Secondary $b\bar{b}$ from $g \to b\bar{b}$ in PS can convert light jet into b-jet
    $\to$ even interpretation changes

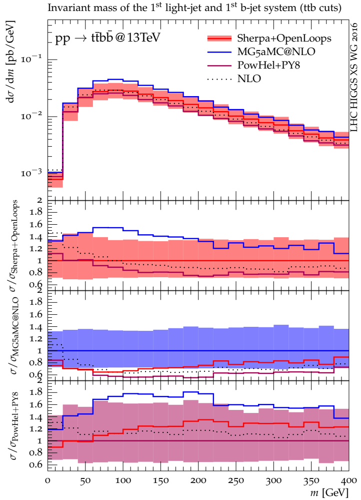




Status of $t\bar{t}b\bar{b}$

- Several tools on the market
  - Sherpa + OpenLoops
    Cascioli, Maierhöfer, Moretti, Pozzorini, Siegert arXiv:1309.5912
  - PowHel + Pythia/Herwig
    [Bevilacqua,Garzelli,Kardos] arXiv:1709.06915
  - PowhegBox + OpenLoops + Pythia/Herwig [Jezo,Lindert,Moretti,Pozzorini]
    arXiv:1802.00426
  - MG5_aMC + Pythia/Herwig
  - Herwig7 + OpenLoops
- History of out-of-the-box comparisons:
  - Large discrepancies
  - Due in part to pQCD uncertainties
  - But also beyond: Parton Shower, NLO+PS matching algorithm
- Ongoing: Tuned comparison
- Fixed-order studies of $t\bar{t}b\bar{b}j$ at NLO show stabilization of K-factor for $\mu_R = (E_{T,t}E_{T,\bar{t}}E_{T,b}E_{T,\bar{b}})^{1/4}$
  $\rightarrow$ New benchmark for NLO+PS programs! [Buccioni,Pozzorini,Zoller '19]

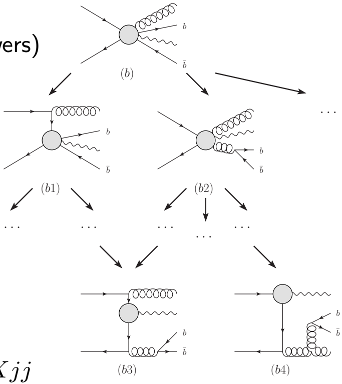
[HXSWG] arXiv:1610.07922



Matching X+jets & $Xb\bar{b}$

[Krause,Siegert,SH] arXiv:1904.09382

- **Interpret $Xb\bar{b}$ as part of Xjj**
  1. Cluster to obtain parton shower history
  2. Apply $\alpha_s(\mu_R^2) \to \alpha_s(p_T^2)$ reweighting
  3. Apply Sudakov factors $\Delta(t, t')$ (trial showers)
- **Remove double-counting**
  1. Cluster PS-level event using inverse PS
  2. Look at leading two emissions
     - Heavy Flavour $\to$ keep from $Xb\bar{b}$ ("direct component")
     - Light Flavour $\to$ keep from X+jets ("fragmentation component")
     - Subleading $g \to b\bar{b}$ splittings not from $Xb\bar{b}$ ME, but $X4j$ ME+PS



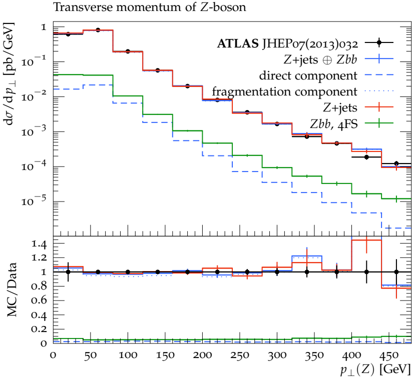
- **Match 5F→4F in PDFs and α_s**
  1. Use 5F PDF / α_s to be consistent with Xjj
  2. Use matching coefficients to correct to 4F scheme
     [Buza,Matiounine,Smith,van Neerven] hep-ph/9612398, [Forte,Napoletano,Ubiali] arXiv:1607.00389
     $\to$ Coefficients up to (N)LL generated by (N)LO parton shower!
  3. Reweighting needed only for α_s in hard ME

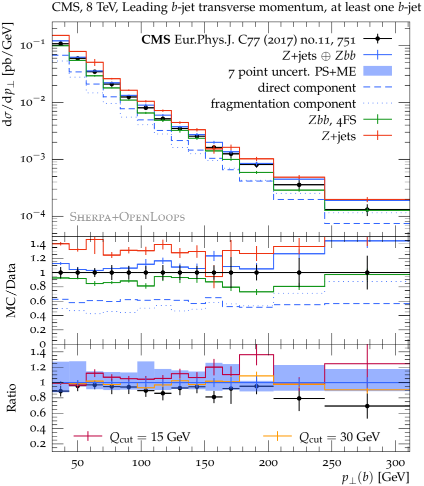
Can be applied to LO and NLO merging!

Example: Z+jets & $Zb\bar{b}$

- Validation with LHC data

[Krause,Siegert,SH] arXiv:1904.09382





	Data [pb]	**Fusing** [pb]
$Z+\geq 1b$	$3.55 \pm 0.24_{\text{comb}}$	$3.80(5) \pm$ 0.83 0.33
$Z+\geq 2b$	$0.331 \pm 0.037_{\text{comb}}$	$0.282(4) \pm$ 0.027 0.022

Matching $t\bar{t}$+jets & $t\bar{t}b\bar{b}$

[Katzy,Krause,Pollard,Siegert] in preparation

- Combination of $t\bar{t}$+0,1j@NLO+2,3j@LO and massive $t\bar{t}b\bar{b}$@NLO
- 2-bjet production dominated by direct component, but 1-bjet observables with equal contributions from direct and fragmentation configurations!

[F.Siegert SM@LHC '19]