Merging Matrix Elements with Parton Showers

Stefan Höche

SLAC NAL Theory Group

MCnet-LPCC School on Event Generators for LHC

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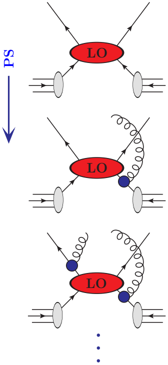
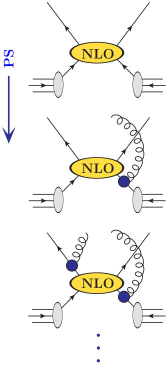
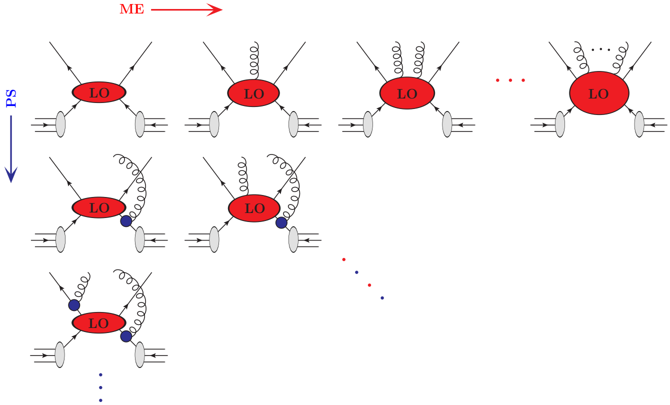
Parton showers (PS)



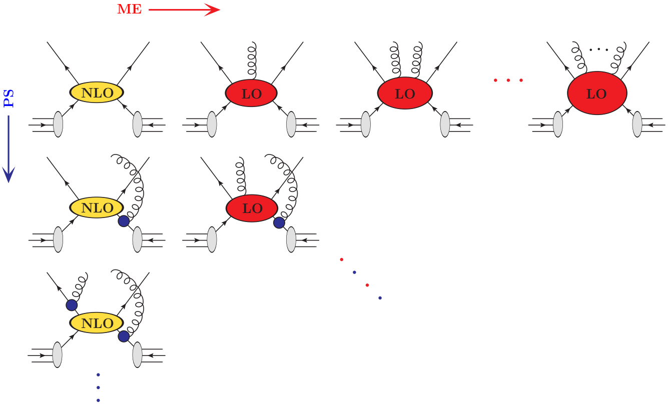
NLO PS matching (NLOPS)



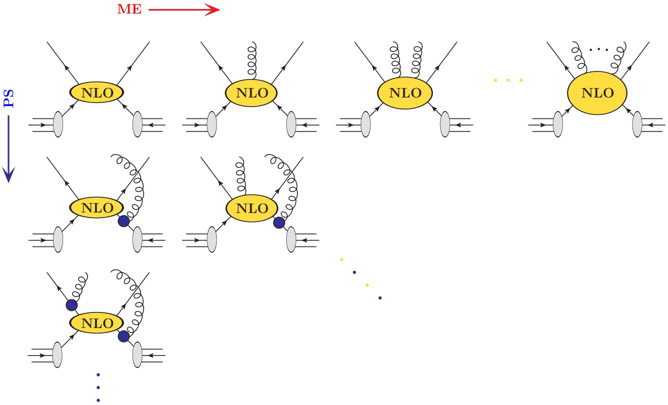
ME PS merging (MEPS)



MEPS combined with NLOPS (MENLOPS)



NLOPS merging with NLOPS (MEPS@NLO)

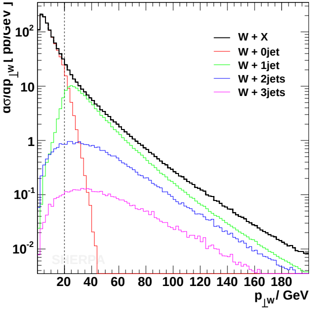


Outline

- General considerations
- LO merging methods
  - CKKW & MLM
  - CKKW-L & METS
- MENLOPS
- NLO merging methods
  - NL^3SP
  - MEPS@NLO

Basic idea of merging

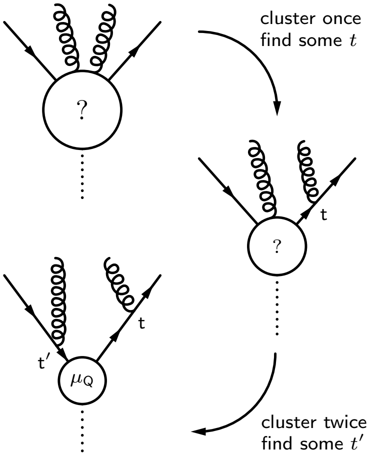
- Separate phase space into "hard" and "soft" region
- Matrix elements populate hard domain
- Parton shower populates soft domain
- Need criterion to define "hard" & "soft" $\rightarrow$ jet measure Q and corresponding cut, $Q_{\rm cut}$



Parton-shower histories

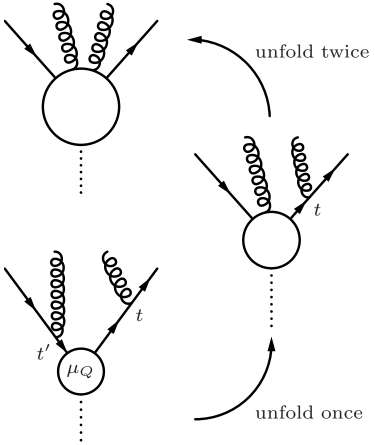
[André,Sjöstrand] hep-ph/9708390

- Start with some "core" process
  for example $e^+e^- \to q\bar{q}$
- This process is considered inclusive
  It sets the resummation scale μ_Q^2
- Higher-multiplicity ME can be reduced to core by clustering
- If we want to match ME & PS the correct clustering algorithm suggests itself
  - Identify most likely splitting according to PS emission probability
  - Combine partons into mother according to PS kinematics
  - Continue until core process



Truncated & vetoed parton showers

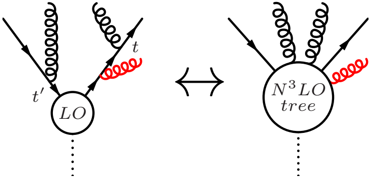
- If higher-multiplicity ME can be clustered back to core that means it is included in the inclusive cross section
- Must compute Sudakov suppression corresponding to no-decay probability of each intermediate parton
  $\rightarrow$ make inclusive ME exclusive
- Here the merging methods differ most



Truncated & vetoed parton showers

Efficient scheme to compute Sudakov suppression: Pseudo-showers

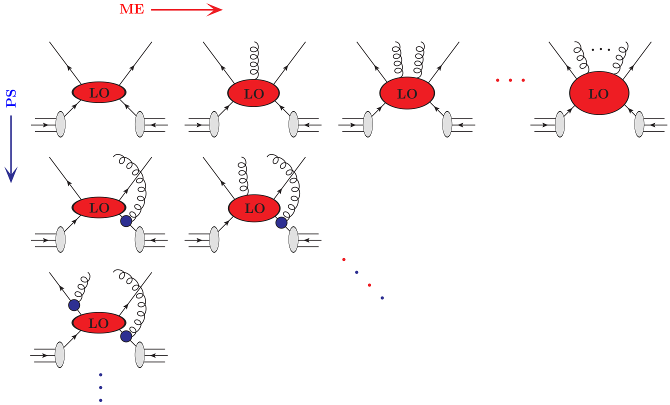
- Start PS from core process
- Evolve until predefined branching
  $\leftrightarrow$ truncated parton shower
- Emissions that would produce additional hard jets lead to event rejection (veto)



This corresponds to computing a Sudakov form factor given by

$$\Delta_n^{(\mathrm{PS})}(t,\mu_Q^2;>Q_{\mathrm{cut}}) = \exp\left\{-\int_t^{\mu_Q^2} \mathrm{d}\Phi_1\, K_n(\Phi_1)\,\Theta(Q-Q_{cut})\right\}$$

ME PS merging (MEPS)



Jet rates in $e^+e^- \to$hadrons

[Catani,Olsson,Turnock,Webber] PLB269(1991)432

- Durham jet measure $\to \min(E_i^2, E_j^2)(1 - \cos\theta_{ij})$
- Jet rates known to NLL

$$R_2(Q_\text{cut}, Q) = \Big[\Delta_q(Q_\text{cut}, Q)\Big]^2$$

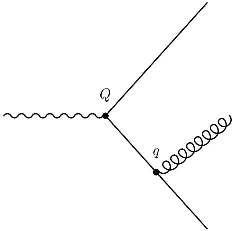
$$R_3(Q_\text{cut}, Q) = 2\,\Delta(Q_\text{cut}, Q)\int_{Q_\text{cut}}^{Q} \mathrm{d}q\,\Gamma_q(q,Q)\,\frac{\Delta_q(Q_\text{cut}, Q)}{\Delta_q(Q_\text{cut}, q)}\,\Delta_q(Q_\text{cut}, q)\Delta_g(Q_\text{cut}, q)$$

$$= 2\Big[\Delta(Q_\text{cut}, Q)\Big]^2\int_{Q_\text{cut}}^{Q} \mathrm{d}q\,\Gamma_q(q,Q)\,\Delta_g(Q_\text{cut}, q)$$

Sudakov form factor and NLL branching probability

$$\Delta_q(Q', Q) = \exp\left(-\int_{Q'}^{Q} \mathrm{d}q\,\Gamma(q,Q)\right)$$

$$\Gamma(q,Q) = \frac{2C_F}{\pi}\frac{\alpha_s(q)}{q}\left(\ln\frac{Q}{q} - \frac{4}{3}\right)$$



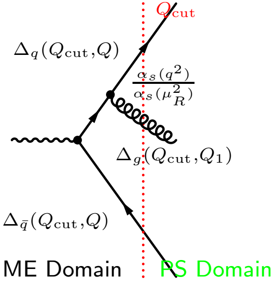
CKKW merging

[Catani,Krauss,Kuhn,Webber] hep-ph/0109231
[Krauss] hep-ph/0205283

- Select ME according to $\sigma(> Q_{\rm cut})$
- Construct PS history
- Weight each vertex with $\alpha_s(q^2)/\alpha_s(\mu_R^2)$
- Weight parton of type i from Q_j to Q_k by

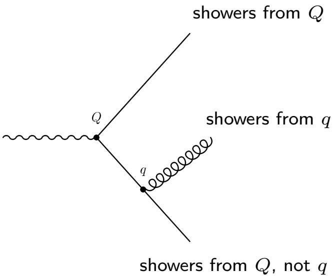
  $\Delta_i(Q_{\rm cut}, Q_j)/\Delta_i(Q_{\rm cut}, Q_k)$

  $\rightarrow$ Sudakov suppression to NLL accuracy
- Veto parton shower if emission has $Q > Q_{\rm cut}$



Practical implementation of CKKW

- Convenient choices made
  - Clustering with Durham k_T-algorithm
  - Analytic Sudakov form factors
  - No truncated showers
    Instead redefined starting scales
- Correct to NLL, but
  exact correspondence with PS is lost
- Problems due to modified colour flow
  and kinematics



MLM merging

[Mangano,Moretti,Pittau] hep-ph/0108069
[Mangano,Moretti,Piccinini,Treccani] hep-ph/0611129

- Define parton-level jets using cone algorithm

$$R_{ij}^2 = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$$
$$E_{T,i} > E_{T,\min}, \qquad R_{ij} > R_{\min}$$

- Generate parton showers from n-jet events
  No Sudakov suppression at this point
- Form new jets on showered final state
- Reject if number of jets increased
  or jets not "matched" to partons in R_{ij}
- Sudakov suppression achieved by jet matching

CKKW-L and METS merging

Algorithms with

- Exact correspondence between clustering & PS evolution
- Sudakov form factors as defined in parton shower

[Lönnblad] hep-ph/0112284
[Lönnblad,Prestel] arXiv:1109.4829

CKKW-L

- Truncated showers generate suppression, but no emissions
- Jet criterion dynamically redefined during PS evolution

[SH,Krauss,Schumann,Siegert] arXiv:0903.1219
[Hamilton,Richardson,Tully] arXiv:0905.3072

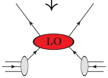
METS

- Truncated parton showers generate emissions and suppression
- Accounts for mismatch between jet criterion and evolution variable

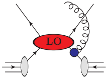
MEPS merging in MC@NLO notation

- Differential event rate to $\mathcal{O}(\alpha_s)$ given by

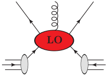
$$\mathrm{d}\sigma_{\mathrm{MEPS}} = \mathrm{d}\Phi_n \mathrm{B}_n(\Phi_n) \Big[\Delta_n^{(\mathrm{PS})}(t_c, \mu_Q^2)$$



$$+ \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1 \mathrm{K}_n(\Phi_1)\, \Delta_n^{(\mathrm{PS})}(t(\Phi_1), \mu_Q^2)\, \Theta(Q_{\mathrm{cut}} - Q) \Big]$$



$$+ \mathrm{d}\Phi_n \int \mathrm{d}\Phi_1\, \mathrm{B}_{n+1}(\Phi_{n+1})\, \Delta_n^{(\mathrm{PS})}(t(\Phi_{n+1}), \mu_Q^2; > Q_{\mathrm{cut}})\, \Theta(Q - Q_{\mathrm{cut}})$$

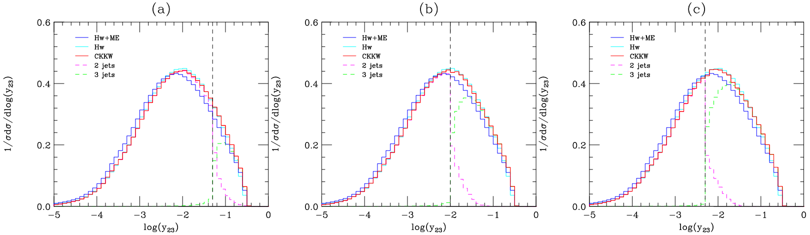


- Jet veto in PS
- Jet cut on $n+1$-parton final state

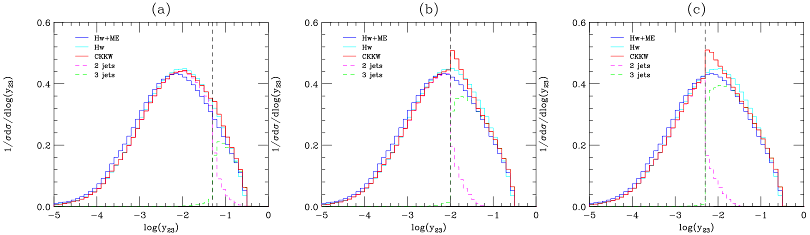
Effect of truncated showers in $e^+e^- \to$ hadrons

[Hamilton, Richardson, Tully] arXiv:0905.3072

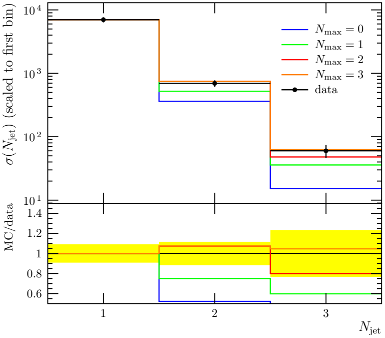
truncated shower on

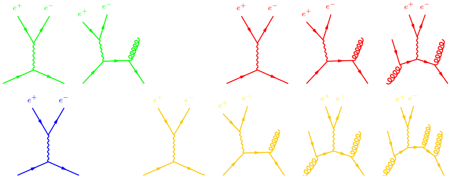


truncated shower off



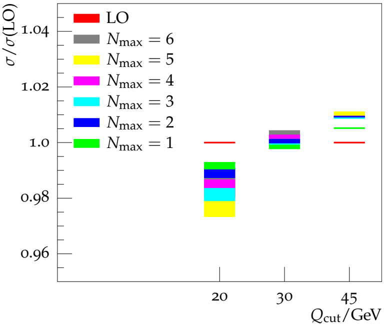
Z+jets at the Tevatron





- MC predictions for exclusive n-jet rates match data well as long as corresponding final states are described by matrix elements

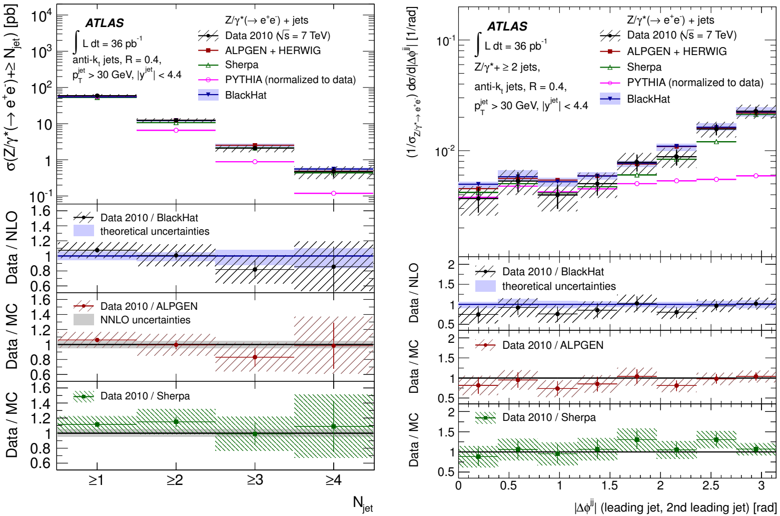
Z+jets at the Tevatron



- MEPS effectively replaces splitting kernels of the parton shower with ratios of LO matrix elements for the emission terms
- We have not corrected the Sudakov form factors, hence there is a mismatch between emission- and no-emission probability
- The inclusive cross section changes, but corrections are small

Z+jets at the LHC

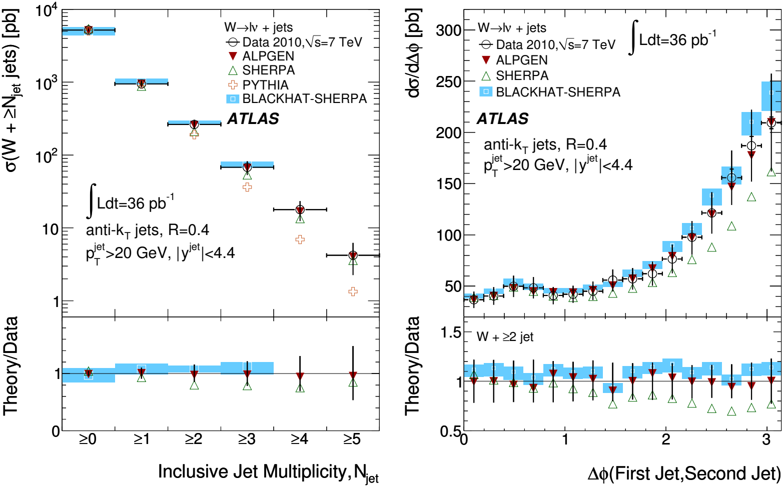
[ATLAS] arXiv:1111.2690



- Good agreement with both ALPGEN (MLM) and Sherpa
- PS alone fails for $n_{\text{jet}} \geq 2$

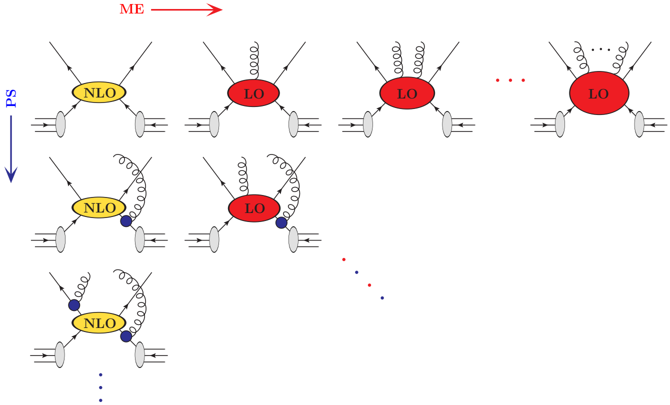
W+jets at the LHC

[ATLAS] arXiv:1201.1267



- Good agreement with ALPGEN (MLM), not so good with Sherpa

MEPS combined with NLOPS (MENLOPS)

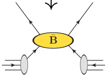


MENLOPS

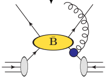
[Hamilton,Nason] arXiv:1004.1764
[SH,Krauss,Schönherr,Siegert] arXiv:1009.1127

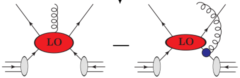
- Increase accuracy below $Q_{\rm cut}$ to full NLO

$$\mathrm{d}\sigma = \mathrm{d}\Phi_n \bar{\mathrm{B}}_n^{(\mathrm{A})}(\Phi_n)\Bigg[\Delta_n^{(\mathrm{A})}(t_c,\mu_Q^2)$$

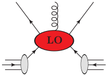


$$+\int_{t_c}^{\mu_Q^2}\mathrm{d}\Phi_1\,\frac{\mathrm{D}_n^{(\mathrm{A})}(\Phi_{n+1})}{\mathrm{B}_n(\Phi_n)}\,\Delta_n^{(\mathrm{A})}(t,\mu_Q^2)\Theta(Q_{\rm cut}-Q)\Bigg] + \mathrm{d}\Phi_n\int\mathrm{d}\Phi_1\,\mathrm{H}_n^{(\mathrm{K})}(\Phi_{n+1})\Theta(Q_{\rm cut}-Q)$$





$$+\,\mathrm{d}\Phi_n\int\mathrm{d}\Phi_1\,\mathrm{k}_n^{(\mathrm{A})}(\Phi_{n+1})\mathrm{B}_{n+1}(\Phi_{n+1})\,\Delta_n^{(\mathrm{PS})}(t,\mu_Q^2;>Q_{\rm cut})\,\Theta(Q-Q_{\rm cut})$$



- Local K-factor for smooth merging

MENLOPS

- Local K-factor for POWHEG

$$\mathrm{k}_n^{(\mathrm{R})}(\Phi_n) = \frac{\bar{\mathrm{B}}_n^{(\mathrm{R})}(\Phi_n)}{\mathrm{B}_n(\Phi_n)}$$

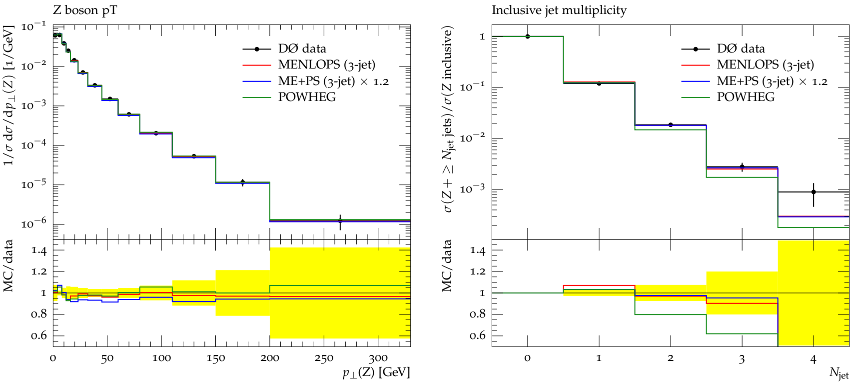
- Local K-factor for MC@NLO

$$\mathrm{k}_n^{(\mathrm{K})}(\Phi_{n+1}) = \frac{\bar{\mathrm{B}}_n^{(\mathrm{K})}(\Phi_n)}{\mathrm{B}_n(\Phi_n)}\left(1 - \frac{\mathrm{H}_n^{(\mathrm{K})}(\Phi_{n+1})}{\mathrm{R}_n(\Phi_{n+1})}\right) + \frac{\mathrm{H}_n^{(\mathrm{K})}(\Phi_{n+1})}{\mathrm{R}_n(\Phi_{n+1})}$$

- Amounts to rescaling higher-multiplicity ME such that their contribution to MENLOPS event sample is the same as their original contribution to the MEPS event sample

Z+jets at Tevatron

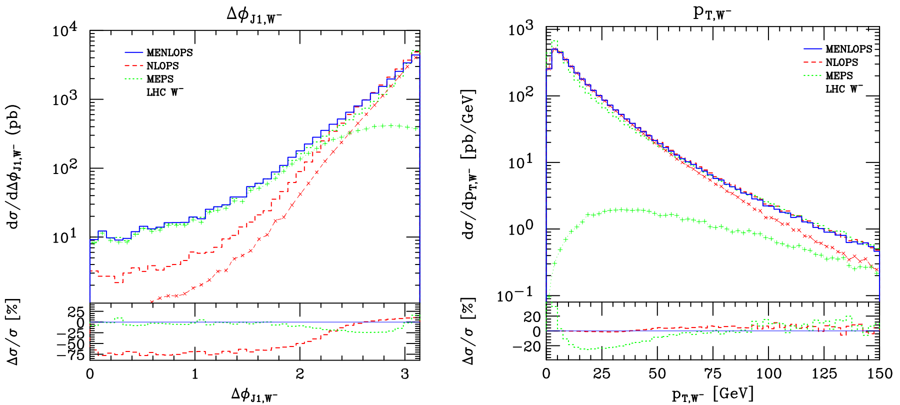
[SH,Krauss,Schönherr,Siegert] arXiv:1009.1127



- MENLOPS, NLOPS & rescaled METS agree for more inclusive observables, e.g. Z-p_T
- Jet rates in MENLOPS improved vs. NLOPS

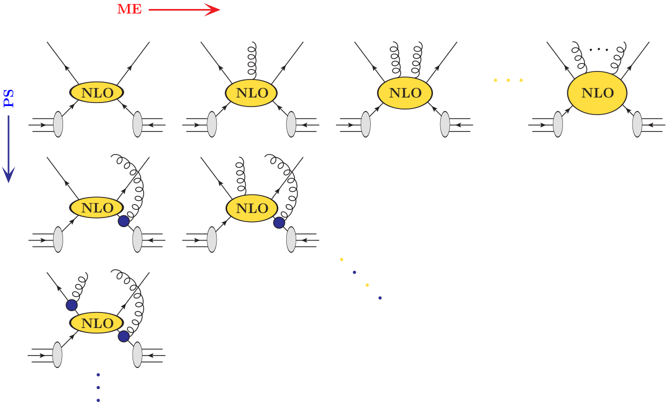
W+jets at LHC

[Hamilton,Nason] arXiv:1004.1764



- MEPS dominates MENLOPS in hard region
- Hard contribution from NLOPS vetoed

NLOPS merging with NLOPS (MEPS@NLO)



NL^{3}SP

[Lavesson,Lönnblad] arXiv:0811.2912
[Lönnblad,Prestel] ICHEP'12

- Ingredients: Rescaled CKKW-L plus POWHEG sample
  Global K-factor for CKKW-L $\to K = 1 + \sum \alpha_s^i(\mu_R^2) k_i$
- Change NLO parton-level scales to CKKW-L scheme
  - Renormalization scale $\to$ use 1-loop running of α_s

$$\alpha_s(\mu_R^2) \to \alpha_s(\mu_R^2)\left(1 - \frac{\alpha_s(\mu_R^2)}{2\pi}\beta_0 \log\frac{t}{\mu_R^2}\right)$$

  - Factorization scale $\to$ use DGLAP evolution of PDFs

$$f_a(x,Q^2) \to f_a(x,Q^2) + \frac{\alpha_s(\mu_R^2)}{2\pi}\log\frac{t}{\mu_F^2}\sum_{b=q,g}\int_x^1 \frac{\mathrm{d}z}{z}\,P_{ab}(z)\,f_b(x/z,\mu_F^2)$$

- Remove $1 + \mathcal{O}(\alpha_s)$ term present in NLO-scaled CKKW-L sample

$$K\Delta_n^{(\mathrm{PS})}(t_c,\mu_Q^2;>Q_{\mathrm{cut}}) - \Big(1 + \alpha_s(\mu_R^2)\,k_1\Big) + \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1 \frac{\alpha_s(\mu_R^2)}{\alpha_s(t)}\mathrm{K}_n\,\Theta(Q_{n+1} - Q_{\mathrm{cut}})$$

NL^{3}SP

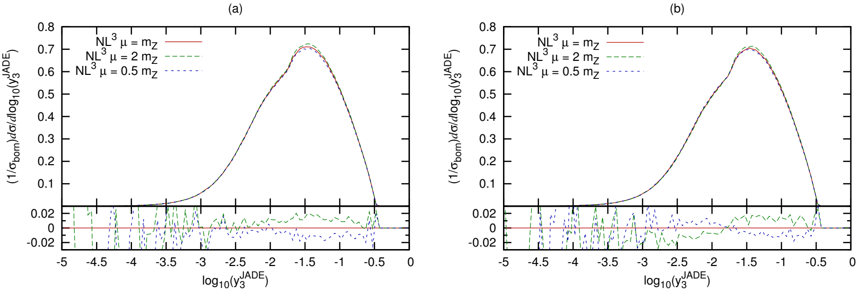
- Differential event rate for exclusive $n+k$-jet events assuming all scales already chosen correct in POWHEG

$$
\begin{aligned}
\mathrm{d}\sigma^{n+k,\mathrm{excl}}_{\mathrm{NL^3SP}} =&\ \mathrm{d}\Phi_{n+k}\,\Theta(Q_{n+k}-Q_{\mathrm{cut}})\prod_{i=n}^{n+k-1}\Delta^{(\mathrm{PS})}_i(t_{i+1},t_i;<Q_{\mathrm{cut}})\\
&\times\Bigg\{\bar{\mathrm{B}}^{(\mathrm{R})}_{n+k}\left[\Delta^{(\mathrm{R})}_{n+k}(t_c,t_{n+k})+\int_{t_c}^{t_{n+k}}\mathrm{d}\Phi_1\,\frac{\mathrm{R}_{n+k}}{\mathrm{B}_{n+k}}\,\Delta^{(\mathrm{R})}_{n+k}(t,t_n)\,\Theta(Q_{\mathrm{cut}}-Q_{n+k+1})\right]\\
&\quad+\mathrm{B}_{n+k}\Bigg(K\prod_{i=n}^{n+k-1}\Delta^{(\mathrm{PS})}_i(t_{i+1},t_i;>Q_{\mathrm{cut}})\\
&\qquad\qquad-\left(1+\alpha_s(\mu_R^2)\,k_1\right)+\sum_{i=n}^{n+k}\int_{i+1}^{i}\mathrm{d}\Phi_1\,\mathrm{K}_i\,\Theta(Q_{i+1}-Q_{\mathrm{cut}})\Bigg)\Bigg\}
\end{aligned}
$$

- If emission above t_{n+k}, but below Q_{cut}, reject event $\rightarrow$ PS domain
- Subtraction needed only in ME / PS overlap region

$e^+e^- \to$hadrons at LEP

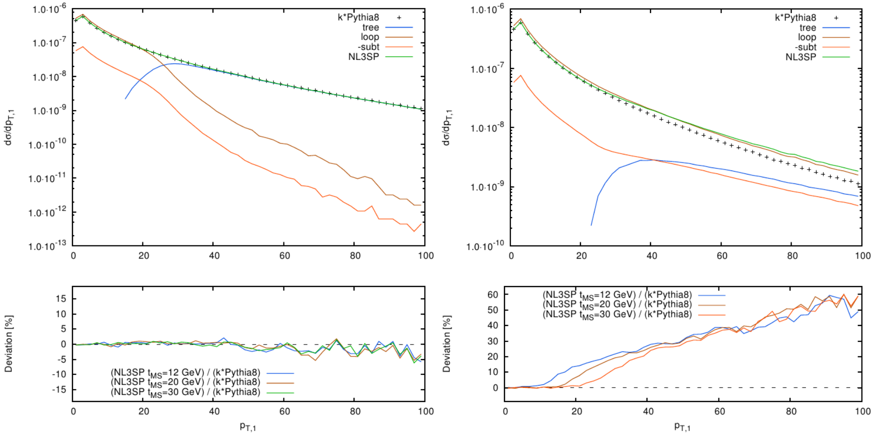
[Lavesson,Lönnblad] arXiv:0811.2912



- Scale variations around 2%
- Agreement between 1- and 2-loop
  but no furter reduction of uncertainty

W+jets at LHC

[Lönnblad,Prestel] ICHEP'12



- Compare W+0-jet at NLO $\leftrightarrow$ W+0,1-jet at NLO

MEPS@NLO

[SH,Krauss,Schönherr,Siegert] arXiv:1207.5030
[Gehrmann,SH,Krauss,Schönherr,Siegert] arXiv:1207.5031

- Define compound evolution kernel

$$\tilde{\mathrm{D}}^{(\mathrm{A})}_{n+k}(\Phi_{n+k+1}) = \mathrm{D}^{(\mathrm{A})}_{n+k}(\Phi_{n+k+1})\,\Theta(t_{n+k}-t_{n+k+1}) + \mathrm{B}_{n+k}(\Phi_{n+k}) \sum_{i=n}^{n+k-1} \mathrm{K}_i(\Phi_i)\,\Theta(t_i - t_{n+k+1})\Theta(t_{n+k+1}-t_{i+1})$$

- Extend MC@NLO modified subtraction

$$\tilde{\mathrm{B}}^{(\mathrm{A})}_{n+k}(\Phi_{n+k}) = \Big[\mathrm{B}_{n+k}(\Phi_{n+k}) + \tilde{\mathrm{V}}_{n+k}(\Phi_{n+k}) + \mathrm{I}_{n+k}(\Phi_{n+k})\Big] + \int \mathrm{d}\Phi_1 \Big[\tilde{\mathrm{D}}^{(\mathrm{A})}_{n+k}(\Phi_{n+k+1}) - \mathrm{S}_{n+k}(\Phi_{n+k+1})\Big]$$

$$\tilde{\mathrm{H}}^{(\mathrm{A})}_{n+k}(\Phi_{n+k+1}) = \mathrm{R}_{n+k}(\Phi_{n+k+1}) - \tilde{\mathrm{D}}^{(\mathrm{A})}_{n+k}(\Phi_{n+k+1})$$

- Scales of NLO calculation chosen in accordance with MEPS

MEPS@NLO

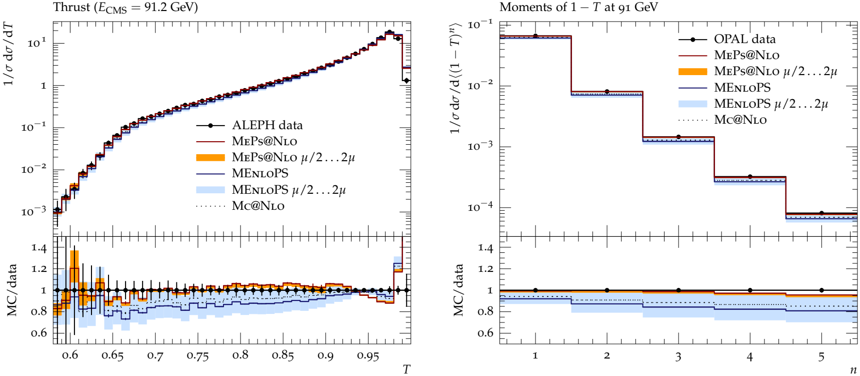
- Differential event rate for exclusive $n+k$-jet events

$$
\begin{aligned}
\mathrm{d}\sigma^{n+k,\mathrm{excl}}_{\mathrm{MEPS@NLO}} &= \mathrm{d}\Phi_{n+k}\,\Theta(Q(\Phi_{n+k})-Q_{\mathrm{cut}})\,\tilde{\mathrm{B}}^{(\mathrm{A})}_{n+k} \\
&\times\left[\tilde{\Delta}^{(\mathrm{A})}_{n+k}(t_c,\mu_Q^2)+\int_{t_c}^{\mu_Q^2}\mathrm{d}\Phi_1\,\frac{\tilde{\mathrm{D}}^{(\mathrm{A})}_{n+k}}{\mathrm{B}_{n+k}}\,\tilde{\Delta}^{(\mathrm{A})}_{n+k}(t,\mu_Q^2)\,\Theta(Q_{\mathsf{cut}}-Q_{n+k+1})\right] \\
&+\int\mathrm{d}\Phi_{n+k+1}\,\tilde{\mathrm{H}}^{(\mathrm{A})}_{n+k}(\Phi_{n+k+1})\,\tilde{\Delta}^{(\mathrm{PS})}_{n+k}(t_{n+k+1},\mu_Q^2;>Q_{\mathrm{cut}})\,\Theta(Q_{\mathsf{cut}}-Q(\Phi_{n+k+1}))
\end{aligned}
$$

- Structurally equivalent to MENLOPS
- Truncated PS contributes at $\mathcal{O}(\alpha_s)$

$e^+e^- \to$hadrons at LEP

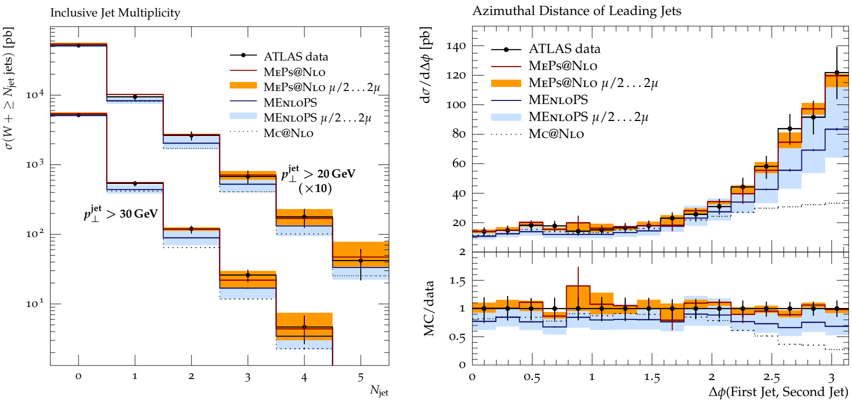
[Gehrmann,SH,Krauss,Schönherr,Siegert] arXiv:1207.5031



- MEPS@NLO with 2,3&4 jet PL at NLO plus 5&6 jet PL at LO
- MENLOPS with 2-6 jet PL at LO

W+jets at LHC

[SH,Krauss,Schönherr,Siegert] arXiv:1207.5030



- MEPS@NLO with 0,1&2 jet PL at NLO plus 3&4 jet PL at LO
- MENLOPS with 0-4 jet PL at LO

Summary

- Merging matrix elements and parton showers requires
  - Identification of ME with PS branching history
  - Computation of Sudakov form factors which make MEs exclusive
- Practical implementations of MEPS merging differ mostly in
  - How the PS history is defined
  - How Sudakov form factors are calculated
- Truncated vetoed parton showers useful to efficiently compute Sudakov form factors
- First ideas for promoting MEPS merging to NLO

MEPS with dynamic Q_{cut}

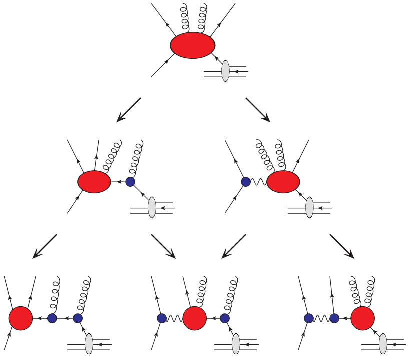
[Carli,Gehrmann,SH] arXiv:0912.3715

- Example: deep-inelastic scattering
- Virtual γ preferentially at $Q^2 \to 0$ but jets often have $k_T^2 > Q^2$
- PS will not capture this situation as $\mu_Q^2 = Q^2 \sim 0 \to$ no phase space
- Need to identify dynamic cut as

$$\frac{1}{Q_{\text{cut}}^2} = \frac{1}{\bar{Q}_{\text{cut}}^2} + \frac{1}{S_{\text{DIS}}^2 Q^2}$$

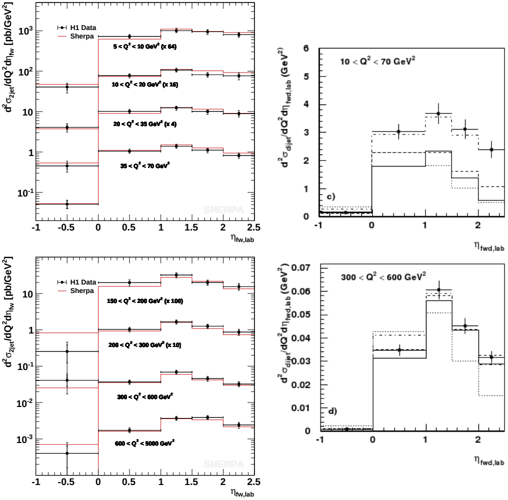
Jet either hard by itself ($k_T > \bar{Q}_{\text{cut}}$) or hard compared to γ ($k_T \gtrsim S_{\text{DIS}} Q$)

- soft γ & hard jet $\to$ $\gamma p \to jj$ "core" process
- soft γ & two hard jets $\to$ $pg \to jj$ / $pq \to jj$ "core"



MEPS with dynamic $Q_{\rm cut}$

[Carli,Gehrmann,SH] arXiv:0912.3715



- More inclusive predictions lead to good agreement with data