

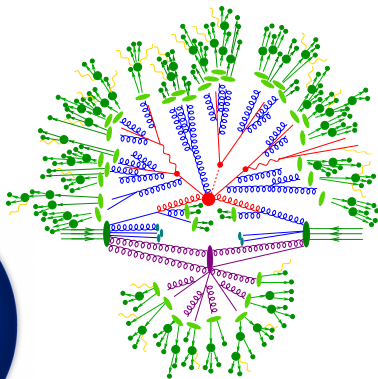
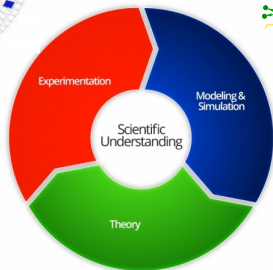
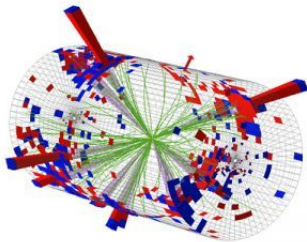
# Precision QCD simulations for the LHC

Stefan Höche

SLAC National Accelerator Laboratory

HEP Monday Seminar

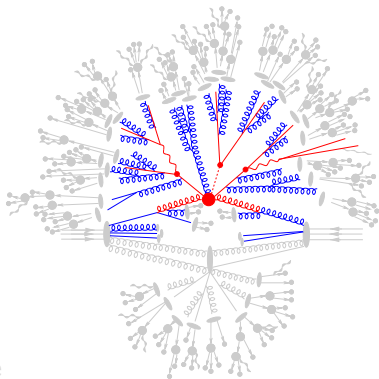
Caltech, 04/27/2015



$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\Psi}\not{D}\Psi + h.c.$$

## Aspects of the theory

- ▶ Perturbative QCD
  - ▶ **Hard processes**
  - ▶ **Radiative corrections**
- ▶ Non-perturbative QCD
  - ▶ Hadronization
  - ▶ Particle decays



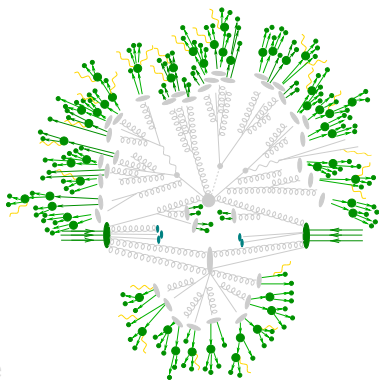
## Divide et Impera

- ▶ Quantity of interest: Total interaction rate
- ▶ Convolution of short & long distance physics

$$\sigma_{p_1 p_2 \rightarrow X} = \sum_{i,j \in \{q,g\}} \int dx_1 dx_2 \underbrace{f_{p_1,i}(x_1, \mu_F^2) f_{p_2,j}(x_2, \mu_F^2)}_{\text{long distance}} \underbrace{\hat{\sigma}_{ij \rightarrow X}(x_1 x_2, \mu_F^2)}_{\text{short distance}}$$

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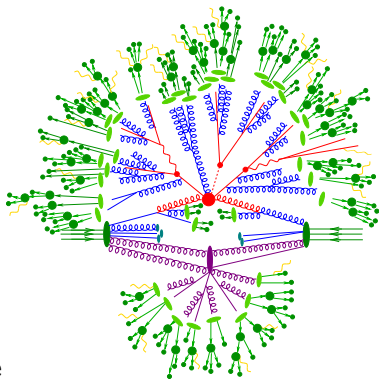
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## All processes of interest

- ▶ Parton shower Monte Carlo (Herwig, Pythia, Sherpa, ...)
- ▶ Automated tree-level calculations & merging with PS (AlpGen, CompHEP, Helac, MadGraph, Sherpa, ...)
- ▶ Automated NLO virtual corrections (BlackHat, GoSam, Helac, MadLoop, MadGolem, NJet, OpenLoops, ...)
- ▶ Matching to parton shower (aMC@NLO, POWHEG Box, Sherpa, ...)
- ▶ Merging at NLO (aMC@NLO, Pythia, Sherpa, ...)

## Selected processes

- ▶ Inclusive NNNLO ( $gg \rightarrow H$ )
- ▶ Inclusive NNLO (jets,  $H + \text{jet}$ ,  $W + \text{jet}$ , single top, ...)
- ▶ Differential NNLO ( $W, Z, gg \rightarrow H, t\bar{t}, V\gamma, VV, VH, \dots$ )
- ▶ NNLO +  $N^x$ LL resummation ( $e^+e^- \rightarrow 2/3$  jets,  $gg \rightarrow H, \dots$ )
- ▶ NNLO matching to PS ( $W, Z, gg \rightarrow H$ )

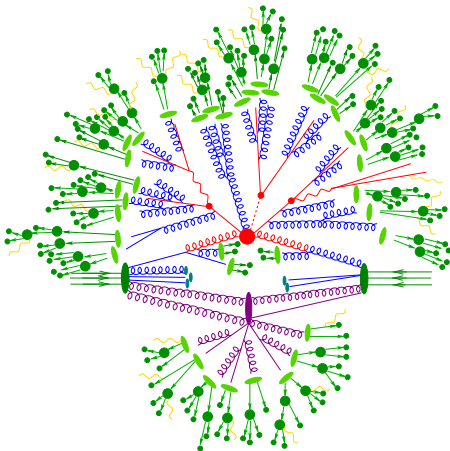
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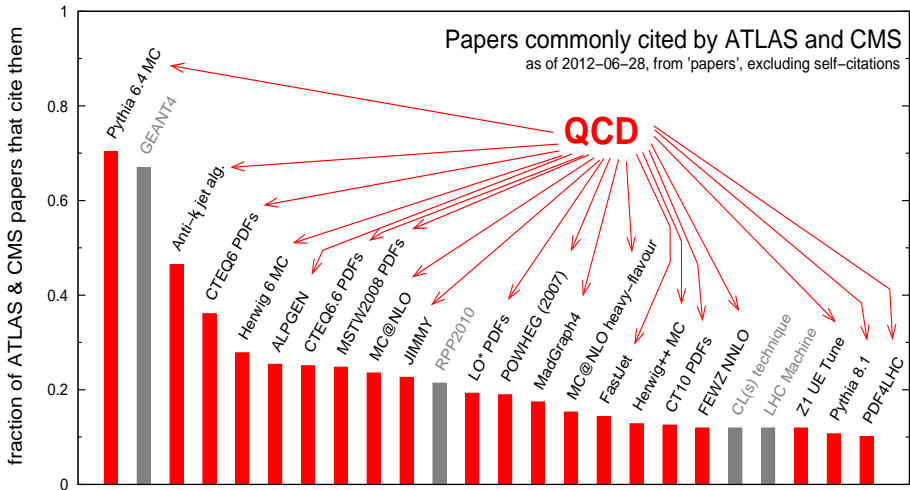
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- ▶ Inclusive NNNLO ( $gg \rightarrow H$ )
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- ▶ Differential NNLO ( $W,Z,gg \rightarrow H,t\bar{t},V\gamma,VV,VH,\dots$ )
- ▶ NNLO+N<sup>x</sup>LL resummation ( $e^+e^- \rightarrow 2/3$  jets,  $gg \rightarrow H,\dots$ )
- ▶ NNLO matching to PS ( $W,Z,gg \rightarrow H$ )

1. **Matrix Element (ME) generators**  
simulate “hard” part of scattering
2. **Parton Showers (PS)**  
produce Bremsstrahlung
3. **Multiple interaction models**  
simulate “secondary” interactions
4. **Fragmentation models**  
“hadronize” QCD partons
5. **Hadron decay packages**  
simulate unstable hadron decay
6. **YFS generators**  
produce QED Bremsstrahlung

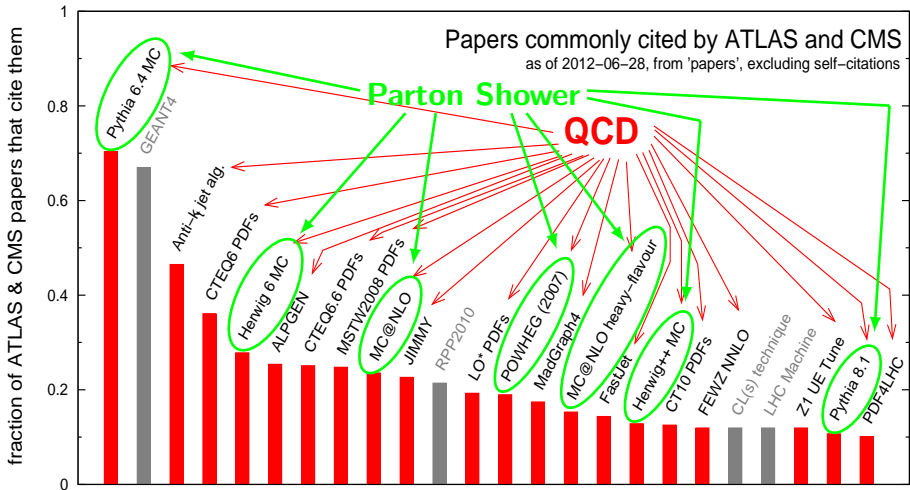






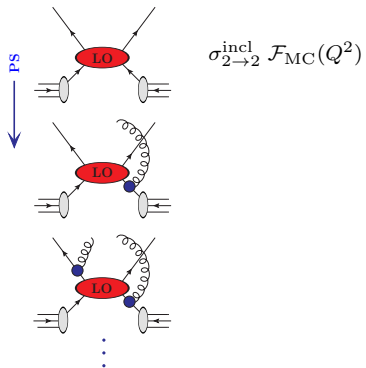
Plot by GP Salam based on data from ATLAS, CMS and INSPIREHEP

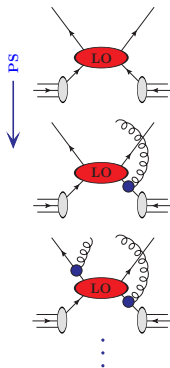
[G.P. Salam, La Thuile 2012]



Plot by GP Salam based on data from ATLAS, CMS and INSPIREHEP

[G.P. Salam, La Thuile 2012]



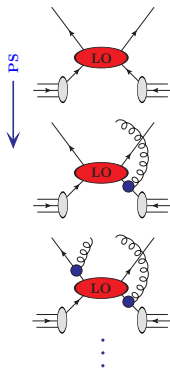


$$\sigma_{2 \rightarrow 2}^{\text{incl}} \left[ \Delta(t_c, Q^2) \right.$$

$$+ \int_{t_c}^{Q^2} \frac{dt}{t} \int dz \frac{\alpha_s}{2\pi} P(z) \Delta(t, Q^2)$$

$$+ \frac{1}{2} \left( \int_{t_c}^{Q^2} \frac{dt}{t} \int dz \frac{\alpha_s}{2\pi} P(z) \right)^2 \Delta(t, Q^2)$$

+ ...

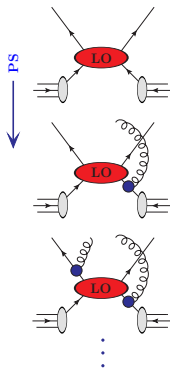


$$\sigma_{2 \rightarrow 2}^{\text{incl}} \left[ \Delta(\tau_c) \right]$$

$$+ \int_{\tau_c}^1 \frac{d\tau}{\tau} \int dz \frac{\alpha_s}{2\pi} P(z) \Delta(\tau)$$

$$+ \frac{1}{2} \left( \int_{\tau_c}^1 \frac{d\tau}{\tau} \int dz \frac{\alpha_s}{2\pi} P(z) \right)^2 \Delta(\tau)$$

+ ...

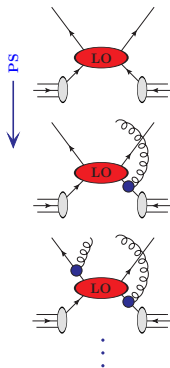


$$\sigma_{2 \rightarrow 2}^{\text{incl}} \left[ \Delta(\tau_c) \right]$$

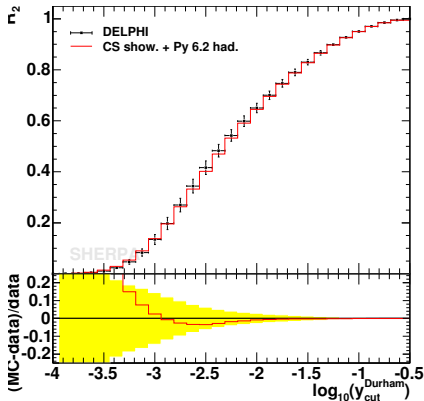
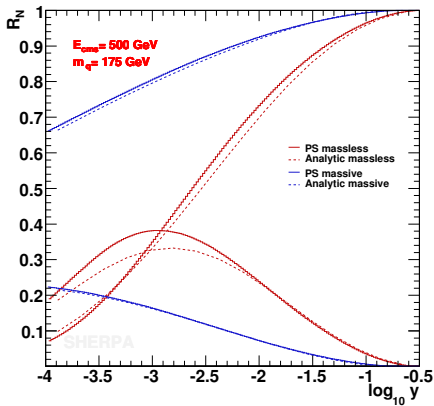
$$+ \int_{\tau_c}^1 d\tau \frac{\alpha_s}{\tau} (A \log \tau + B) \Delta(\tau)$$

$$+ \frac{1}{2} \left( \int_{\tau_c}^1 d\tau \frac{\alpha_s}{\tau} (A \log \tau + B) \right)^2 \Delta(\tau)$$

+ ...



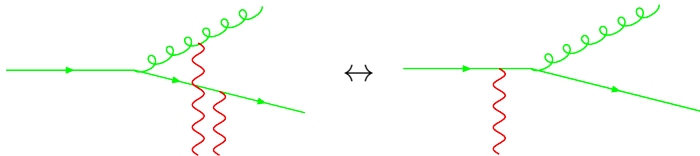
$$\begin{aligned}
 \sigma_{2 \rightarrow 2}^{\text{incl}} & \left[ \exp \left\{ - \int_{\tau_c}^1 d\tau' \frac{\alpha_s}{\tau'} (A \log \tau' + B) \right\} \right. \\
 & + \int_{\tau_c}^1 d\tau \frac{\alpha_s}{\tau} (A \log \tau + B) \exp \left\{ - \int_{\tau_c}^1 d\tau' \frac{\alpha_s}{\tau'} (A \log \tau' + B) \right\} \\
 & + \frac{1}{2} \left( \int_{\tau_c}^1 d\tau \frac{\alpha_s}{\tau} (A \log \tau + B) \right)^2 \exp \left\{ - \int_{\tau_c}^1 d\tau' \frac{\alpha_s}{\tau'} (A \log \tau' + B) \right\} \\
 & + \dots
 \end{aligned}$$





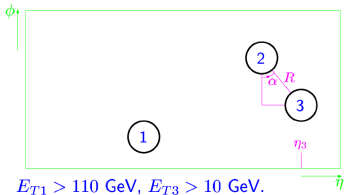
[Marchesini, Webber] NPB310(1988)461

- ▶ Gluons with large wavelength not capable of resolving charges of emitting color dipole individually

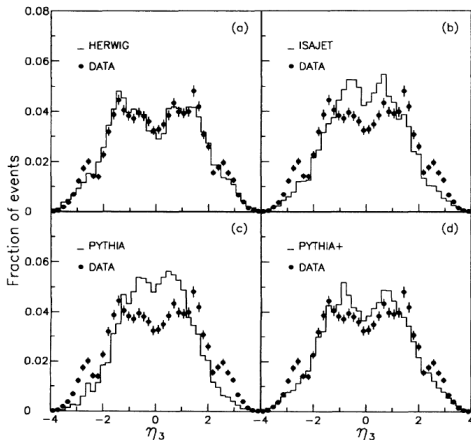


- ▶ Emission occurs with combined charge of mother parton instead
- ▶ Net effect is destructive interference outside cone with opening angle defined by emitting color dipole
- ▶ Soft anomalous dimension  $A$  is halved as a consequence of changed  $z$ -bounds

- ▶ Observed in 3-jet events
- ▶ Purely virtuality ordered PS's produced too much radiation in central region
- ▶ Angular ordered / angular vetoed PS's ok



[CDF] PRD50(1994)5562



[Nagy] hep-ph/0601021 [Schumann,Krauss] arXiv:0709.1027

[Plätzer,Gieseke] arXiv:0909.5593

- ▶ Angular ordered PS does not fill full phase space → prefer alternative solution of coherence problem
- ▶ Coherence from eikonal factors  
Rewrite à la [Catani,Seymour] hep-ph/9605323

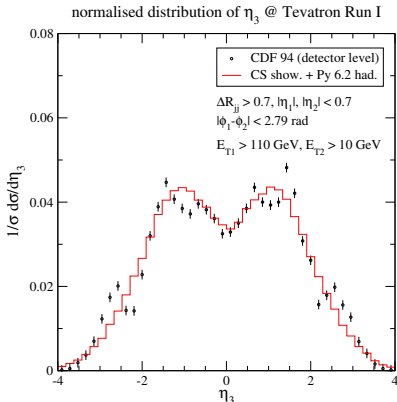
$$\frac{p_i p_k}{(p_i q)(q p_k)} \rightarrow \frac{1}{p_i q} \frac{p_i p_k}{(p_i + p_k) q} + \frac{1}{p_k q} \frac{p_i p_k}{(p_i + p_k) q}$$

- ▶ “Spectator”-dependent PS kernels

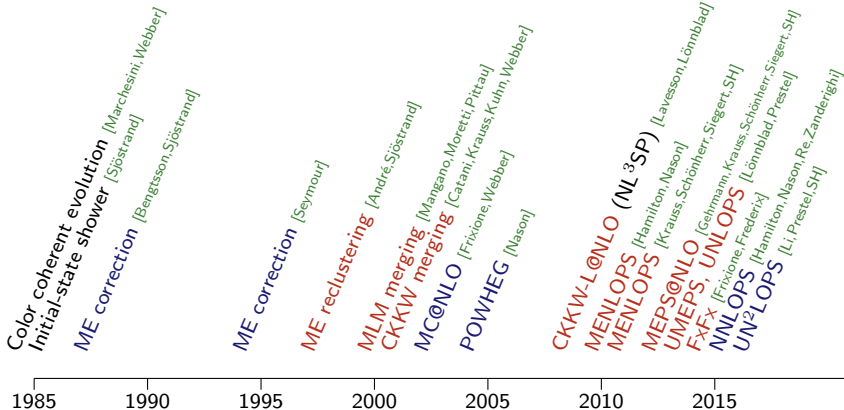
$$\frac{1}{1-z} \rightarrow \frac{1-z+k_{\perp}^2/Q^2}{(1-z)^2+k_{\perp}^2/Q^2}$$

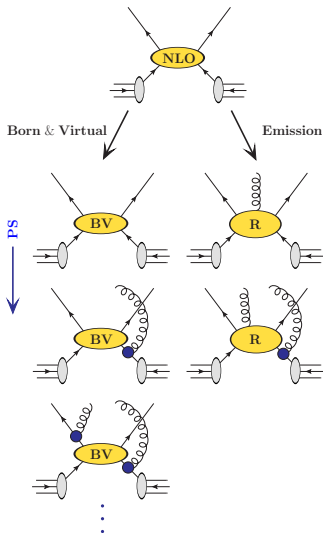
Singular in soft-collinear region only

- ▶ Captures dominant coherence effects (3-parton correlations)
- ▶ Does not account for sub-leading color effects



Merging related  
Matching related





- ▶ Idea: Use real radiative corrections to improve PS approximation

- ▶ Methods: MC@NLO & POWHEG

[Frixione,Webber] hep-ph/0204244 [Nason] hep-ph/0409146

- ▶ Leading-order calculation for observable  $O$

$$\langle O \rangle = \int d\Phi_B B(\Phi_B) O(\Phi_B)$$

- ▶ NLO calculation for same observable

$$\langle O \rangle = \int d\Phi_B \left\{ B(\Phi_B) + \tilde{V}(\Phi_B) \right\} O(\Phi_B) + \int d\Phi_R R(\Phi_R) O(\Phi_R)$$

- ▶ Parton-shower result (until first emission)

$$\langle O \rangle = \int d\Phi_B B(\Phi_B) \mathcal{F}_{\text{MC}}(\mu_Q^2, O)$$

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- ▶ Parton-shower result (until first emission)

$$\langle O \rangle = \int d\Phi_B B(\Phi_B) \left[ \Delta^{(K)}(t_c) O(\Phi_B) + \int_{t_c} d\Phi_1 K(\Phi_1) \Delta^{(K)}(t(\Phi_1)) O(\Phi_R) \right]$$

Phase space:  $d\Phi_1 = dt dz d\phi J(t, z, \phi)$

Splitting functions:  $K(t, z) \rightarrow \alpha_s / (2\pi t) \sum P(z) \Theta(\mu_Q^2 - t)$

Sudakov factors:  $\Delta^{(K)}(t) = \exp \left\{ - \int_t d\Phi_1 K(\Phi_1) \right\}$

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$$\xrightarrow{\mathcal{O}(\alpha_s)} \int d\Phi_B B(\Phi_B) \left\{ 1 - \int_{t_c} d\Phi_1 K(\Phi_1) \right\} O(\Phi_B) + \int_{t_c} d\Phi_B d\Phi_1 B(\Phi_B) K(\Phi_1) O(\Phi_R)$$

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Sudakov factors:  $\Delta^{(K)}(t) = \exp \left\{ - \int_t d\Phi_1 K(\Phi_1) \right\}$



- ▶ Subtract  $\mathcal{O}(\alpha_s)$  PS terms from NLO result ( $t_c \rightarrow 0$ )

$$\langle O \rangle = \int d\Phi_B \left\{ B(\Phi_B) + \tilde{V}(\Phi_B) + B(\Phi_B) \int d\Phi_1 K(\Phi_1) \right\} O(\Phi_B) \\ + \int d\Phi_R \left\{ R(\Phi_R) - B(\Phi_B) K(\Phi_1) \right\} O(\Phi_R)$$

- ▶ In DLL approximation both terms finite  $\rightarrow$   
MC events in two categories, Standard and  $\mathbb{H}$ ard

$$\mathbb{S} \rightarrow \bar{B}^{(K)}(\Phi_B) = B(\Phi_B) + \tilde{V}(\Phi_B) + B(\Phi_B) \int d\Phi_1 K(\Phi_1)$$

$$\mathbb{H} \rightarrow H^{(K)} = R(\Phi_R) - B(\Phi_B) K(\Phi_1)$$

- ▶ Full QCD has color & spin correlations  $\rightarrow$  **NLO subtraction** needed  
 $1/N_c$  corrections faded out in soft region by **smoothing function**

$$\bar{B}^{(K)}(\Phi_B) = B(\Phi_B) + \tilde{V}(\Phi_B) + \mathbf{I}(\Phi_B) + \int d\Phi_1 \left[ \mathbf{S}(\Phi_R) - B(\Phi_B) K(\Phi_1) \right] f(\Phi_1)$$

$$H^{(K)}(\Phi_R) = \left[ R(\Phi_R) - B(\Phi_B) K(\Phi_1) \right] f(\Phi_1)$$

- ▶ Add parton shower, described by generating functional  $\mathcal{F}_{MC}$

$$\langle O \rangle = \int d\Phi_B \bar{B}^{(K)}(\Phi_B) \mathcal{F}_{MC}^{(0)}(\mu_Q^2, O) + \int d\Phi_R H^{(K)}(\Phi_R) \mathcal{F}_{MC}^{(1)}(t(\Phi_R), O)$$

Probability conservation  $\leftrightarrow \mathcal{F}_{MC}(t, 1) = 1$

- ▶ Expansion of matched result until first emission

$$\langle O \rangle = \int d\Phi_B \bar{B}^{(K)}(\Phi_B) \left[ \Delta^{(K)}(t_c) O(\Phi_B) \right. \\ \left. + \int_{t_c} d\Phi_1 K(\Phi_1) \Delta^{(K)}(t(\Phi_1)) O(\Phi_r) \right] + \int d\Phi_R H^{(K)}(\Phi_{n+1}) O(\Phi_R)$$

- ▶ Parametrically  $\mathcal{O}(\alpha_s)$  correct
- ▶ Preserves logarithmic accuracy of PS

## Method 1

[Frixione,Webber] JHEP06(2002)029

- ▶  $f(\Phi_1) \rightarrow 0$  in soft-gluon limit
- ▶ Full NLO only in hard / collinear region  
Missing subleading color terms in soft domain
- ▶ Only affects unresolved gluons  $\rightarrow$  no need to correct in principle

## Method 2

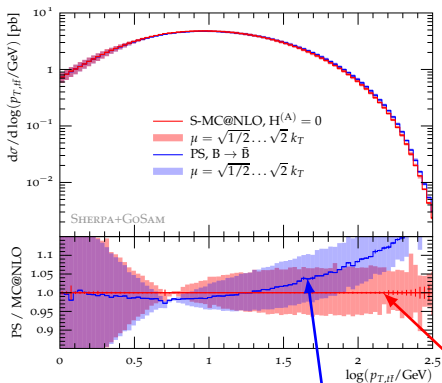
[Krauss,Schönherr,Siegert,SH] JHEP09(2012)049

- ▶ Replace  $B(\Phi_B)K(\Phi_1) \rightarrow S(\Phi_R)$ , i.e. include color & spin correlations
- ▶ May lead to non-probabilistic Sudakov factor  $\Delta^{(S)}(t)$   
Requires modification of veto algorithm
- ▶ Exact cancellation of all divergences without additional smoothing  
Equivalent to one-step full colour parton shower algorithm

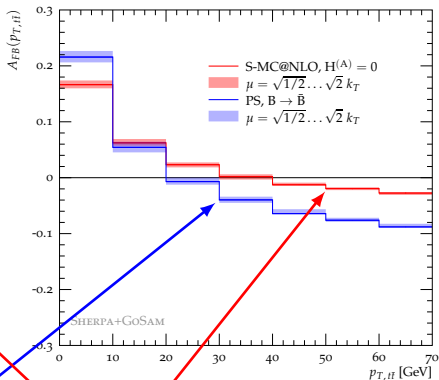
# Does it make a difference?

[Huang,Luisoni,Schönherr,Winter,SH] arXiv:1306.2703

## ▶ Top-quark pair $p_T$

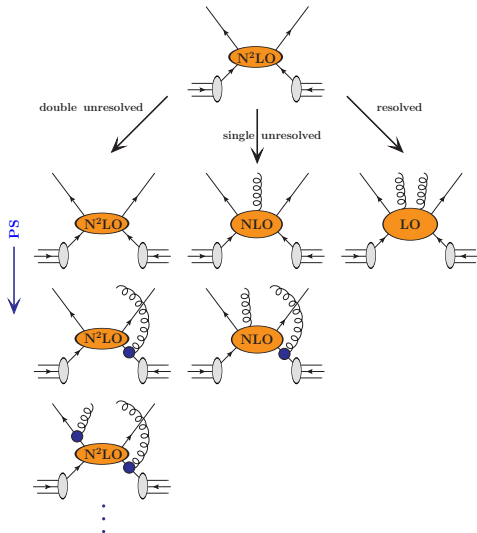


## ▶ Forward-backward asymmetry



leading color

full color (first shower step)



- ▶ PS expression for infrared safe observable,  $O$

$$\langle O \rangle = \int d\Phi_0 B_0 \mathcal{F}_0(\mu_Q^2, O)$$

$$\mathcal{F}_n(t, O) = \Delta_n(t_c, t) O(\Phi_n) + \int_{t_c}^t d\hat{\Phi}_1 K_n \Delta_n(\hat{t}, t) \mathcal{F}_{n+1}(\hat{t}, O)$$

- ▶ **Add ME correction** to first emission ( $B_0 K_0 \rightarrow B_1$ ) & **unitarize**

$$+ \int_{t_c} d\Phi_1 \Delta_0(t_1, \mu_Q^2) B_1 \mathcal{F}_1(t_1, O) - \int_{t_c} d\Phi_1 \Delta_0(t_1, \mu_Q^2) B_1 O(\Phi_0)$$

- ▶ ME evaluated at fixed scales  $\mu_{R/F} \rightarrow$  need to adjust to PS

$$w_1 = \frac{\alpha_s(b t_1)}{\alpha_s(\mu_R^2)} \frac{f_a(x_a, t_1)}{f_a(x_a, \mu_F^2)} \frac{f_{a'}(x_{a'}, \mu_F^2)}{f_{a'}(x_{a'}, t_1)}$$

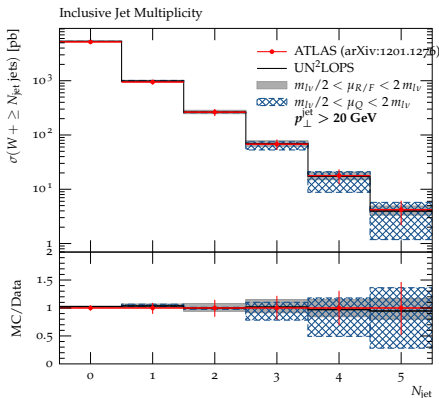
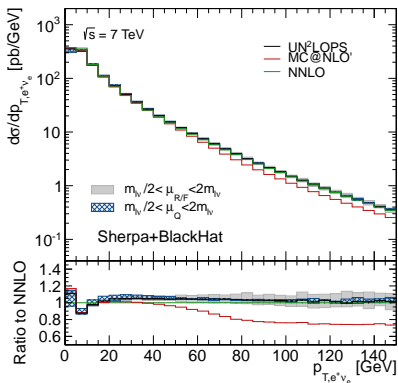
- ▶ Replace  $B_0$  by vetoed xs  $\bar{B}_0^{t_c} = B_0 - \int_{t_c} d\Phi_1 B_1$

$$\langle O \rangle = \left\{ \int d\Phi_0 \bar{B}_0^{t_c} + \int_{t_c} d\Phi_1 \left[ 1 - \Delta_0(t_1, \mu_Q^2) w_1 \right] B_1 \right\} O(\Phi_0) + \int_{t_c} d\Phi_1 \Delta_0(t_1, \mu_Q^2) w_1 B_1 \mathcal{F}_1(t_1, O)$$

- ▶ Promote vetoed cross section to **NNLO**
- ▶ Add NLO corrections to  $B_1$  using **S-MC@NLO**
- ▶ **Subtract**  $\mathcal{O}(\alpha_s)$  term of  $w_1$  and  $\Delta_0$

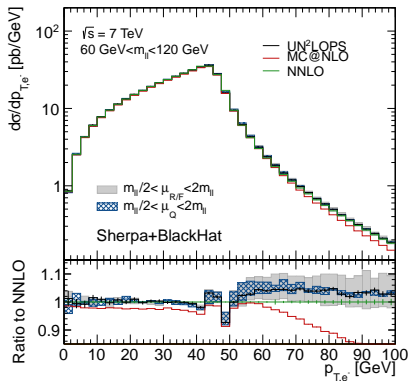
$$\begin{aligned}
 \langle O \rangle = & \int d\Phi_0 \bar{B}_0^{t_c} O(\Phi_0) \\
 & + \int_{t_c} d\Phi_1 \left[ 1 - \Delta_0(t_1, \mu_Q^2) \left( w_1 + w_1^{(1)} + \Delta_0^{(1)}(t_1, \mu_Q^2) \right) \right] B_1 O(\Phi_0) \\
 & + \int_{t_c} d\Phi_1 \Delta_0(t_1, \mu_Q^2) \left( w_1 + w_1^{(1)} + \Delta_0^{(1)}(t_1, \mu_Q^2) \right) B_1 \bar{\mathcal{F}}_1(t_1, O) \\
 & + \int_{t_c} d\Phi_1 \left[ 1 - \Delta_0(t_1, \mu_Q^2) \right] \tilde{B}_1^R O(\Phi_0) + \int_{t_c} d\Phi_1 \Delta_0(t_1, \mu_Q^2) \tilde{B}_1^R \bar{\mathcal{F}}_1(t_1, O) \\
 & + \int_{t_c} d\Phi_2 \left[ 1 - \Delta_0(t_1, \mu_Q^2) \right] H_1^R O(\Phi_0) + \int_{t_c} d\Phi_2 \Delta_0(t_1, \mu_Q^2) H_1^R \mathcal{F}_2(t_2, O) \\
 & + \int_{t_c} d\Phi_2 H_1^E \mathcal{F}_2(t_2, O)
 \end{aligned}$$

- ▶  $\tilde{B}_1^R = \bar{B}_1 - B_1 = \tilde{V}_1 + I_1 + \int d\Phi_{+1} S_1 \Theta(t_2 - t_1)$   
 $H_1^R (H_1^E) \rightarrow$  regular (exceptional) double real configurations

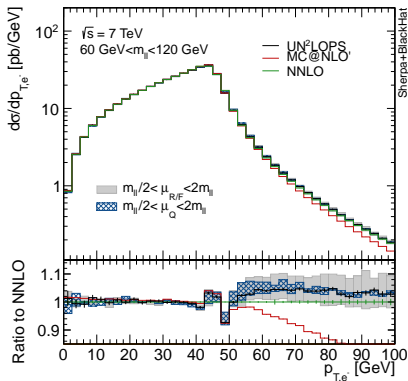


- ▶ Good agreement with S-MC@NLO at low  $p_{T,W}$
- ▶  $W+1\text{-jet}$   $K$ -factor at high  $p_{T,W}$

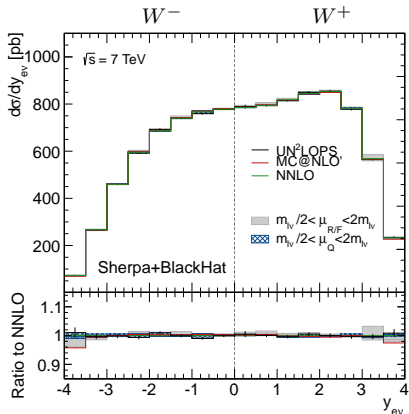
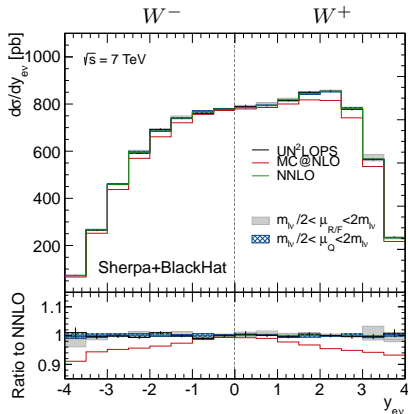




► S-MC@NLO with NLO PDFs



► S-MC@NLO with NNLO PDFs



► S-MC@NLO with NLO PDFs

► S-MC@NLO with NNLO PDFs

- ▶ In soft limit real-emission amplitudes factorize as

$$\begin{aligned}
 |\mathcal{M}_0(1, \dots, j, \dots, n)|^2 &\xrightarrow{j \rightarrow \text{soft}} - \sum_{i, k \neq i} \frac{8\pi\mu^{2\epsilon}\alpha_s}{p_i p_j} \\
 &\times \langle m_0(1, \dots, i, \dots, k, \dots, n) | \frac{\mathbf{T}_i \cdot \mathbf{T}_k p_i p_k}{p_i p_j + p_k p_j} | m_0(1, \dots, i, \dots, k, \dots, n) \rangle .
 \end{aligned}$$

$\mathbf{T}_i$  - color insertion operator for parton  $i$   
 $|m_0(1, \dots, i, \dots, k, \dots, n)\rangle$  - Born amplitude

- ▶ Parton showers use  $\mathbf{T}_i \cdot \mathbf{T}_k \approx -\mathbf{T}_i^2 / \sum_{k \neq i}$
- ▶ Matched shower uses  $\mathbf{T}_i \cdot \mathbf{T}_k$  in first emission
- ▶ Full matrix exponentiation is work in progress  
 Comparison to analytic resummation is a starting point

[Banfi,Salam,Zanderighi] hep-ph/0407286, arXiv:1001.4082

- ▶ Generic NLL resummation framework exists (CAESAR)
- ▶ Observable dependence parametrized as

$$V(\{\tilde{p}\}; k) = d_l \left( \frac{k_t^{(l)}}{Q} \right)^a e^{-b_l \eta^{(l)}} g_l(\phi^{(l)})$$

- ▶ Resummed integrated spectrum for  $V(\{\tilde{p}\}; k) < v$  given by

$$\frac{1}{\sigma} \int_0^v \frac{d^2\sigma}{d\mathcal{B}dv'} dv' = \sum_{\delta \in \text{partonics}} \frac{d\sigma_0^{(\delta)}}{d\mathcal{B}} e^{Lg_1^{(\delta)}(\alpha_s L) + g_2^{(\delta, \mathcal{B})}(\alpha_s L)} [1 + \mathcal{O}(\alpha_s)], \quad L = \log \frac{1}{v}$$

- ▶ LL / NLL coefficients  $g_1$  and  $g_2$  arise from 1- and 2-emission integrals
- ▶  $g_2$  depends on soft function  $\mathcal{S}$  through

$$\log \mathcal{S}(T(L/a)), \quad \text{where} \quad T(L) = \frac{1}{\pi\beta_0} \log \frac{1}{1 - 2\alpha_s\beta_0 L}$$

[Gerwick,SH,Marzani,Schumann] arXiv:1411.7325

- ▶ Soft function known analytically for low-multiplicity final states
- ▶ Generic structure in terms of anomalous dimension  $\Gamma$  is

$$\mathcal{S}(\xi) = \frac{\langle m_0 | e^{-\frac{\xi}{2}\Gamma^\dagger} e^{-\frac{\xi}{2}\Gamma} | m_0 \rangle}{\langle m_0 | m_0 \rangle}, \quad \Gamma = -2 \sum_{i < j} \mathbf{T}_i \cdot \mathbf{T}_j \log \frac{Q_{ij}}{Q_{12}} + i\pi \sum_{i,j=II,FF} \mathbf{T}_i \cdot \mathbf{T}_j$$

- ▶ Insertion of color projectors  $|c_\alpha\rangle\langle c^\alpha|$  leads to matrix structure

$$\mathcal{S}(\xi) = \frac{c_{\alpha\beta} H^{\gamma\sigma} \mathcal{G}_{\gamma\rho}^\dagger c^{\rho\beta} c^{\alpha\delta} \mathcal{G}_{\delta\sigma}}{c_{\alpha\beta} H^{\alpha\beta}}, \quad \mathcal{G}_{\alpha\beta}(\xi) = c_{\alpha\gamma} \exp\left(-\frac{\xi}{2} c^{\gamma\delta} \Gamma_{\delta\beta}\right)$$

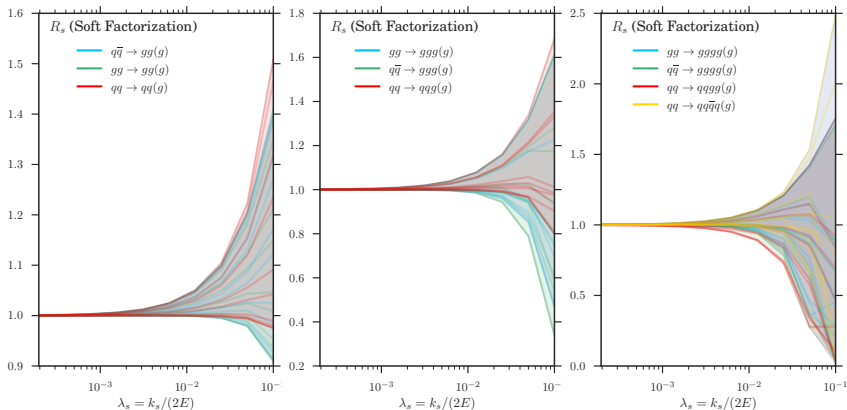
where  $H^{\alpha\beta} = \langle m_0 | c^\alpha \rangle \langle c^\beta | m_0 \rangle$  and  $\Gamma_{\alpha\beta} = \langle c_\alpha | \Gamma | c_\beta \rangle$

- ▶  $c_{\alpha\beta} = \langle c_\alpha | c_\beta \rangle$  - color “metric”,  $H^{\alpha\beta}$  - hard matrix
- ▶ Much effort in the literature is spent on choosing orthogonal bases

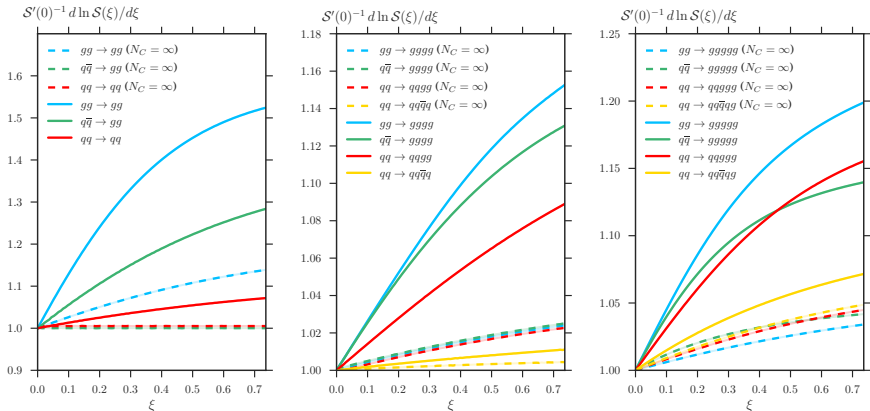
[Sjödahl] arXiv:0906.1121, arXiv:1211.2099, [Keppeler] arXiv:1207.0609

[Gerwick,SH,Marzani,Schumann] arXiv:1411.7325

- ▶ Missing ingredients for resummation at higher multiplicity
  - ▶ **Hard matrix** → ME generator Comix
  - ▶ **Soft anomalous dimension** → Mathematica scripts
- ▶ Remaining problems
  - ▶ **Non-orthogonality of color bases**  
Solved by incorporation of inverse metric  $c^{\alpha\beta} = (c_{\alpha\beta})^{-1}$
  - ▶  **$N_c = 3$  pathologies in overcomplete color bases**  
Solved by numeric matrix inversion at  $N_c = 3 + \varepsilon$



- ▶ Ratio of sum-over-dipole dressed Born to exact matrix elements
- ▶ Checks correctness of soft anomalous dimension and color metric



- Size of sub-leading color contributions for “circle kinematics” (all outgoing partons at  $\eta = 0$ , equally spaced in  $\Delta\phi$ )



[Banfi,Salam,Zanderighi] hep-ph/0407286, arXiv:1001.4082

- ▶ Resummed integrated spectrum for  $V(\{\bar{p}\}; k) < v$  given by

$$\frac{1}{\sigma} \int_0^v \frac{d^2\sigma}{d\mathcal{B}dv'} dv' = \sum_{\delta \in \text{partonics}} \frac{d\sigma_0^{(\delta)}}{d\mathcal{B}} e^{Lg_1^{(\delta)}(\alpha_s L) + g_2^{(\delta, \mathcal{B})}(\alpha_s L)} [1 + \mathcal{O}(\alpha_s)]$$

- ▶ Expansion to NLO leads to LL and NLL coefficients

$$G_{12} = - \sum_{l=1}^n \frac{C_l}{a(a+b_l)}$$

$$G_{11} = - \left[ \sum_{l=1}^n C_l \left( \frac{B_l}{a+b_l} + \frac{1}{a(a+b_l)} \left( \ln \bar{d}_l - b_l \ln \frac{2E_l}{Q} \right) + \frac{1}{a} \ln \frac{Q_{12}}{Q} \right) + \frac{1}{a} \frac{\text{Re}[\Gamma_{\alpha\beta}] H^{\alpha\beta}}{c_{\alpha\beta} H^{\alpha\beta}} + \sum_{l=1}^{n_{\text{initial}}} \frac{\int_{x_l}^1 \frac{dz}{z} P_{lk}^{(0)} \left( \frac{x_l}{z} \right) q^{(k)}(z, \mu_F^2)}{2(a+b_l)q^{(l)}(x_l, \mu_F^2)} \right].$$

- ▶ Missing ingredient for resummation at higher multiplicity
  - ▶ **Generic matching method** → Quasi-local subtraction

[Gerwick,SH,Marzani,Schumann] arXiv:1411.7325

- Compare soft factorization with Catani-Seymour dipole factorization

$$\mathcal{D}_{ij,k}(1, \dots, n) = -\frac{1}{2p_i p_j} \times \langle m_0(1, \dots, ij, \dots, k, \dots, n) | \frac{\mathbf{T}_i \cdot \mathbf{T}_k}{\mathbf{T}_{ij}^2} \hat{V}_{ij,k}(z, k_T, \varepsilon) | m_0(1, \dots, ij, \dots, k, \dots, n) \rangle .$$

- Obtain  $\text{Re}[\Gamma_{\alpha\beta}] H^{\alpha\beta} / c_{\alpha\beta} H^{\alpha\beta}$  from replacement

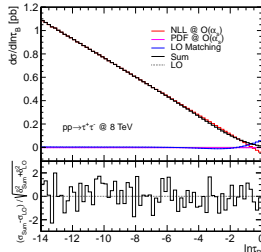
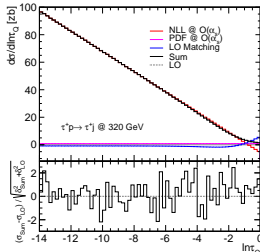
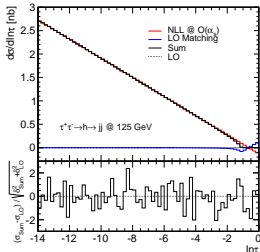
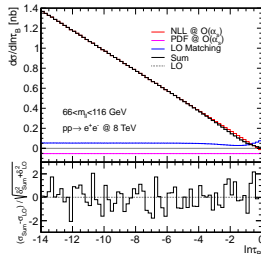
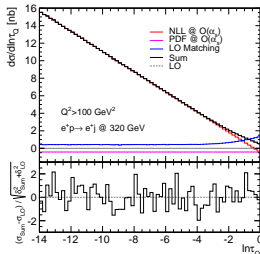
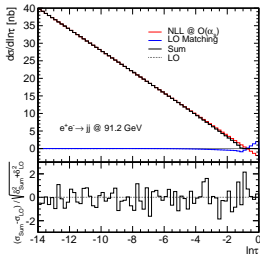
$$\hat{V}_{ij,k}(z, k_T, \varepsilon) \rightarrow \log Q_{(ij)k} / Q_{12}, \text{ and rescaling by } 1/a$$

- Obtain  $G_{12}$  and  $B_l$ -dependent term in  $G_{11}$  from replacement

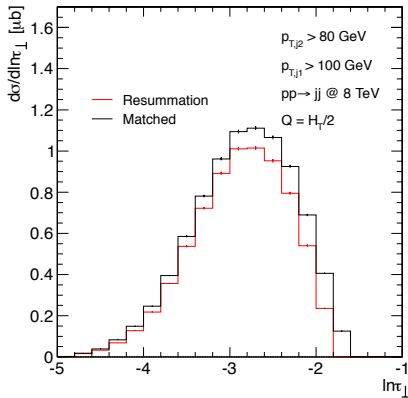
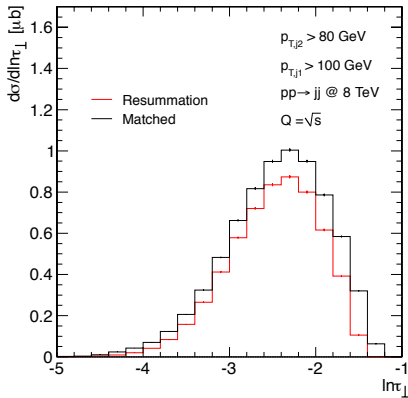
$$\hat{V}_{ij,k} \rightarrow P_{ij,i}, \text{ restricting LL terms to } z^a > v, \text{ and rescaling by } 1/(a + b_l)$$

- Need to identify observable with CS phase space variables:

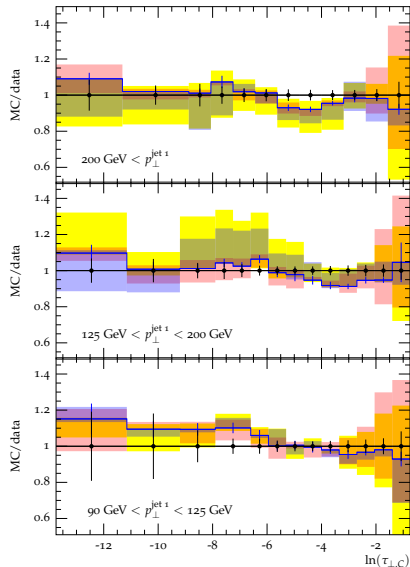
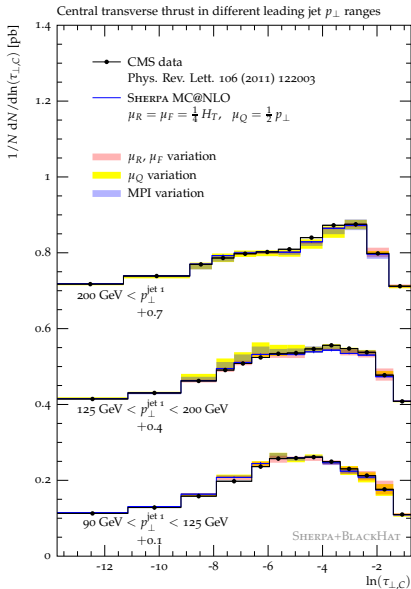
$$v = \begin{cases} y_{ij,k} & \text{FF dipoles} \\ \frac{1 - x_{ij,a}}{1 - x_B} & \text{FI dipoles} \\ u_i & \text{IF dipoles} \\ \frac{v_i}{1 - x_B} & \text{II dipoles} \end{cases}, \quad z = \begin{cases} \tilde{z}_j \text{ or } \tilde{z}_i & \text{FF dipoles} \\ \tilde{z}_j \text{ or } \tilde{z}_i & \text{FI dipoles} \\ \frac{1 - x_{ik,a}}{1 - x_B} & \text{IF dipoles} \\ \frac{1 - x_{i,ab}}{1 - x_B} & \text{II dipoles} \end{cases} .$$



[Gerwick,SH,Marzani,Schumann 2014]

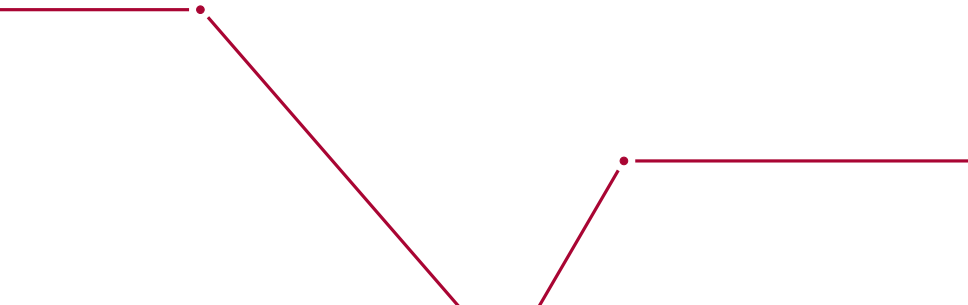


► Full result (NLL resummed and matched) for transverse thrust in  $pp \rightarrow jj$



- ▶ Parton showers are indispensable tools for
  - ▶ phenomenology
  - ▶ experimental analysis
  - ▶ experiment design
- ▶ Matching at (N)NLO & merging at (N)LO can improve PS approximation at fixed jet multiplicity
- ▶ Genuine reduction of uncertainties can only be achieved by improved resummation
- ▶ New tool for NLL resummation at high multiplicity will allow detailed study of sub-leading color effects
- ▶ Another interesting topic under investigation is incorporation of sub-leading logarithms
- ▶ All this is ongoing work, stay tuned!

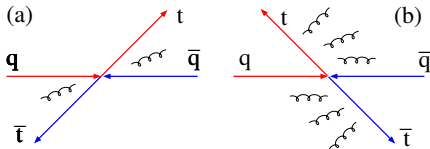
**Thank you for your attention!**



[Skands, Webber, Winter] arXiv:1205.1466

[Huang, Luisoni, Schönherr, Winter, SH] arXiv:1306.2703

- ▶ Parton-shower unitarity broken by splitting of emission phase space
- ▶ Events with  $\Delta y_{t\bar{t}} > 0$  have fewer phase space for radiation



- ▶ But inclusive asymmetry is mainly generated by momentum mapping

$$\Delta\sigma_{+-} = -2 \int \underbrace{d\sigma_{LO} |_{\Delta y > 0} (1 - \Delta_+) P_{+-}}_{\text{subdominant as } \Delta_- < \Delta_+ \text{ ((b) vs. (a))}} + 2 \int \underbrace{d\sigma_{LO} |_{\Delta y < 0} (1 - \Delta_-) P_{-+}}_{\text{dominant as } \Delta_+ > \Delta_- \text{ ((a) vs. (b))}}$$

$P_{-+}/P_{+-}$  - probabilities for  $\Delta y$  to increase / decrease in splitting

- ▶ Dipole showers generate positive rapidity shift in each emission

$$\Delta y_t = \frac{1}{2} \ln \left( 1 + \frac{p_q p_g}{p_q p_t} \left( \frac{1-z}{z} + \frac{m_t^2}{p_q p_t} \right) \frac{\tilde{p}_q^+}{\tilde{p}_t^+} \right) > 0$$

Similar finding for any dipole-like recoil scheme  $\rightarrow$  positive asymmetry