

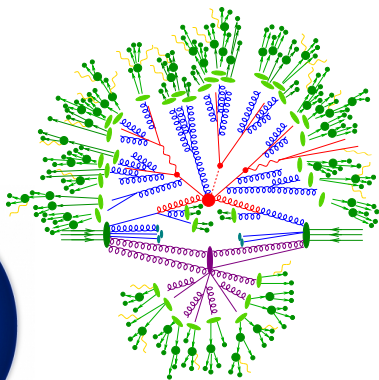
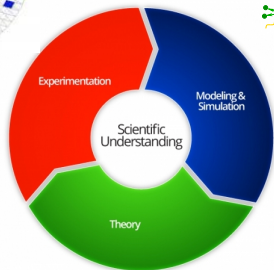
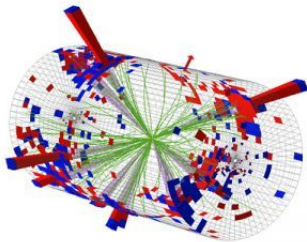
# Precision QCD at the LHC

Stefan Höche

SLAC National Accelerator Laboratory

APS Meeting

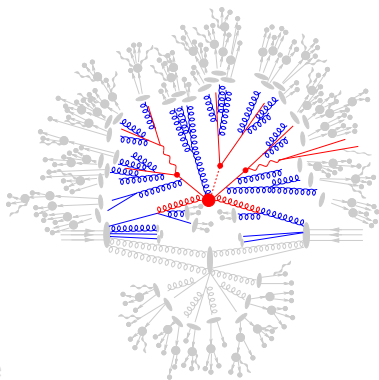
April 13, 2015



$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\Psi}\not{D}\Psi + h.c.$$

## Aspects of the theory

- ▶ Perturbative regime
  - ▶ **Hard processes**
  - ▶ **Radiative corrections**
- ▶ Non-perturbative regime
  - ▶ Hadronization
  - ▶ Particle decays



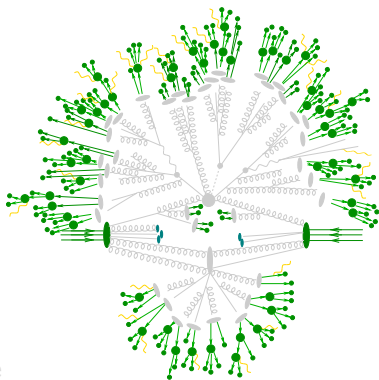
## Divide et Impera

- ▶ Quantity of interest: Total interaction rate
- ▶ Convolution of short & long distance physics

$$\sigma_{p_1 p_2 \rightarrow X} = \sum_{i,j \in \{q,g\}} \int dx_1 dx_2 \underbrace{f_{p_1,i}(x_1, \mu_F^2) f_{p_2,j}(x_2, \mu_F^2)}_{\text{long distance}} \underbrace{\hat{\sigma}_{ij \rightarrow X}(x_1, x_2, \mu_F^2)}_{\text{short distance}}$$

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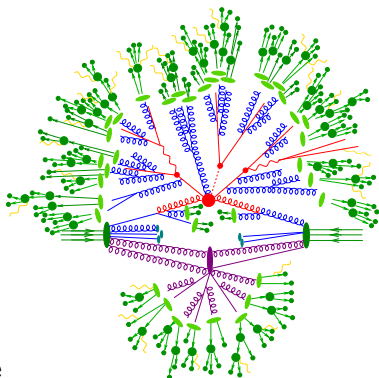
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## All processes of interest

- ▶ Parton shower Monte Carlo (Herwig, Pythia, Sherpa, ...)
- ▶ Automated tree-level calculations & merging with PS (AlpGen, CompHEP, Helac, MadGraph, Sherpa, ...)
- ▶ Automated NLO virtual corrections (BlackHat, GoSam, Helac, MadLoop, MadGolem, NJet, OpenLoops, ...)
- ▶ Matching to parton shower (aMC@NLO, Herwig, POWHEG Box, Sherpa, ...)
- ▶ Merging at LO & NLO (AlpGen, aMC@NLO, Helac, Pythia, Sherpa, ...)

## Selected processes

- ▶ Inclusive NNNLO ( $gg \rightarrow H$ )
- ▶ Inclusive NNLO ( $t\bar{t}$ , jets,  $H + \text{jet}$ ,  $W + \text{jet}$ , single top)
- ▶ Differential NNLO ( $W, Z, gg \rightarrow H, V\gamma, VV, VH$ )
- ▶ NNLO +  $N^{\times}LL$  resummation ( $e^+e^- \rightarrow 2/3 \text{ jets}$ ,  $gg \rightarrow H$ )
- ▶ NNLO + PS ( $W, Z, gg \rightarrow H$ )

- ▶ General approach: subtraction methods

$$d\hat{\sigma}_{\text{NLO}} = \int_{\Phi_n} \left( d\hat{\sigma}^{\text{B}} + d\hat{\sigma}^{\text{V}} + d\hat{\sigma}^{\text{MF}} + \underbrace{\int_{\Phi_1} d\hat{\sigma}^{\text{S}}}_{\text{finite, compute with MC}} \right) + \int_{\Phi_{n+1}} \underbrace{\left( d\hat{\sigma}^{\text{R}} - d\hat{\sigma}^{\text{S}} \right)}_{\text{finite, compute with MC}}$$

- ▶ Universal infrared behavior of amplitudes

- ▶ FKS subtraction [Frixione,Kunszt,Signer 1995]
- ▶ Dipole subtraction [Catani,Seymour 1996 +Dittmaier,Trocsanyi 2002]
- ▶ Antenna subtraction [Kosower 1997]

- ▶ Realized in tree-level ME generators & stand-alone codes

- ▶ Sherpa [Gleisberg,Krauss 2007]
- ▶ MadDipole [Frederix,Greiner,Gehrmann 2008]
- ▶ Helac [Czakon,Papadopoulos,Worek 2009]
- ▶ TeVJet [Seymour,Tevlin 2008]
- ▶ AutoDipole [Hasegawa,Moch,Uwer 2008]
- ▶ MadFKS [Frederix,Frixione,Maltoni,Stelzer 2009]

- ▶ One-loop amplitudes evaluated by extracting coefficients of box/triangle/bubble/tadpole master integrals

$$A = \sum d_i \text{[box diagram]} + \sum c_i \text{[triangle diagram]} + \sum b_i \text{[bubble diagram]} + R$$

- ▶ “Feynmanian” approach → Improved decomposition & reduction

[Denner,Dittmaier 2005] [Binoth,Guillet,Pilon,Heinrich,Schubert 2005]

- ▶ “Unitarian” approach → Use multi-particle cuts & complex momenta

[Bern,Dixon,Dunbar,Kosower 1994] [Britto,Cachazo,Feng 2004] [Ossola,Papadopoulos,Pittau 2006]  
[Forde 2007] [Ellis,Giele,Kunszt,Melnikov 2008]

- ▶ Plethora of (semi-)automated programs: BlackHat, GoSam, HelacNLO, MadLoop, MadGolem, NJet, OpenLoops, . . .

[Badger,Bern,Bevilacqua,Biedermann,Binoth,Cascioli,Cullen,Czakon,Dixon,Ellis, Febres Cordero,Frederix,Frixione,Garzelli,Giele,Goncalves Netto,Greiner,Guffanti, Guillet,vanHameren,Heinrich,Hirschi,Ita,Kardos,Karg,Kauer,Kosower,Lopez-Val,Kunszt, Luisoni,Maierhöfer,Maître,Maltoni,Mastrolia,Mawatari,Melnikov,Ossola,Ozeren, Papadopoulos,Pittau,Plehn,Pozzorini,Reiter,Reuter,Tramontano,Uwer,Wigmore,Worek, Yundin,Zanderighi,Zeppenfeld,...]



[Bern,Dixon,Febres Cordero,SH,Ita,Kosower,Maître,Ozeren 2014]

- ▶ W+jets at 7 TeV,  $E_T^e > 20$  GeV,  $|\eta^e| < 2.5$ ,  $\cancel{E}_T > 20$  GeV  
 $p_T^j > 25$  GeV,  $|\eta^j| < 3$ ,  $M_T^W > 20$  GeV

Jets	$\frac{W^- + (n+1)}{W^- + n}$		$\frac{W^+ + (n+1)}{W^+ + n}$	
	LO	NLO	LO	NLO
1	0.2949(0.0003)	0.238(0.001)	0.3119(0.0005)	0.242(0.002)
2	0.2511(0.0005)	0.220(0.001)	0.2671(0.0004)	0.235(0.002)
3	0.2345(0.0008)	0.211(0.003)	0.2490(0.0005)	0.225(0.003)
4	0.218(0.001)	0.200(0.006)	0.2319(0.0008)	0.218(0.006)

- ▶ Fit to straight line gives (for  $n \geq 2$ )

$$R_{n/(n-1)}^{\text{NLO}, W^-} = 0.248 \pm 0.008 - (0.009 \pm 0.002) n$$

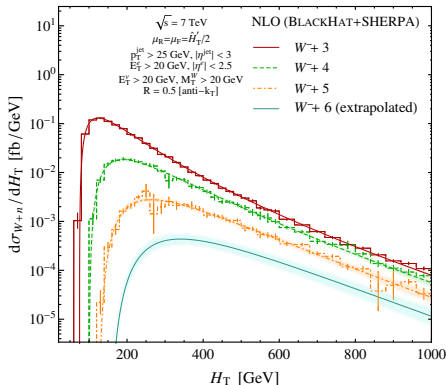
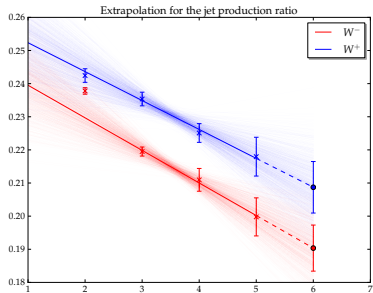
$$R_{n/(n-1)}^{\text{NLO}, W^+} = 0.263 \pm 0.009 - (0.009 \pm 0.003) n$$

- ▶ Extrapolate to six jets

$$W^- + 6 \text{ jets} : 0.15 \pm 0.01 \text{ pb}$$

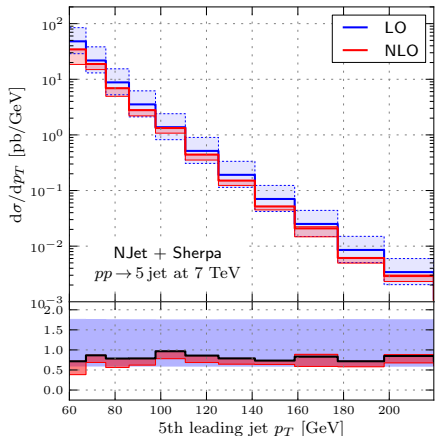
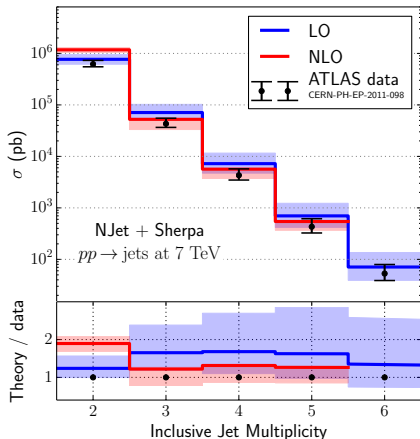
$$W^+ + 6 \text{ jets} : 0.30 \pm 0.03 \text{ pb}$$

[Bern,Dixon,Febres Cordero,SH,Ita,Kosower,Maître 2015]



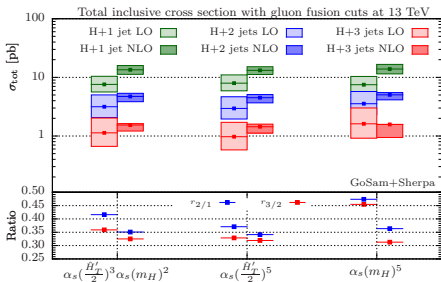
- ▶ Extrapolation of jet rate ratio and  $H_T$  spectrum
- ▶ Scaling proven by jet calculus [Gerwick,Gripiaios,Schumann,Webber 2013]

[Badger, Biedermann, Uwer, Yundin 2013]

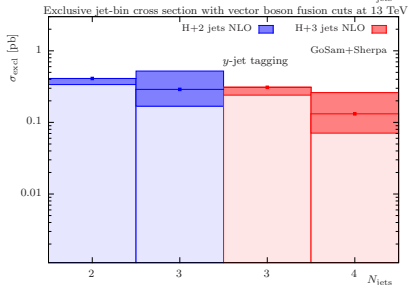
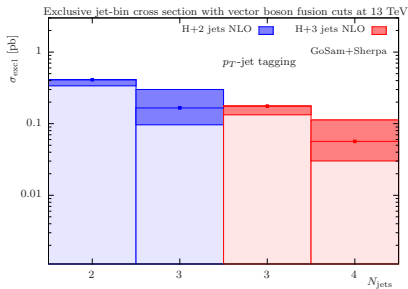


- ▶ Used to understand jet scaling in BSM searches
- ▶ Helps constrain PDFs with LHC data

[Greiner,SH,Luisoni,Schönherr,Winter,Yundin 2015]



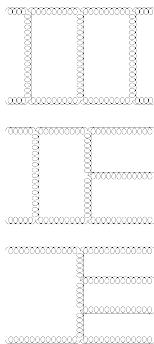
- ▶  $H + 2$  jets through gluon fusion is irreducible background to VBF  $\rightarrow$  get handle on jet veto efficiency through  $H + 3$  jets at NLO
- ▶ Jet scaling in process with topology similar to Drell-Yan



- ▶ Structure of the calculation

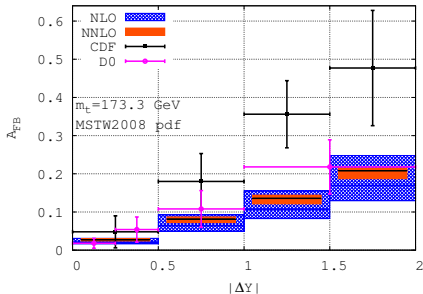
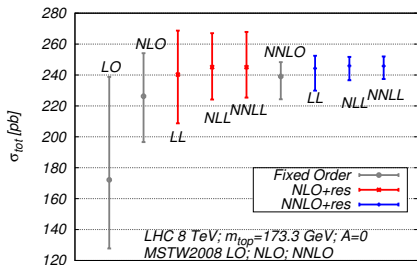
$$\begin{aligned}
 d\hat{\sigma}_{\text{NNLO}} = & \int_{\Phi_{n+2}} \left( d\hat{\sigma}^{RR} - d\hat{\sigma}^S \right) + \int_{\Phi_{n+1}} \left( d\hat{\sigma}^{RV} - d\hat{\sigma}^{VS} + d\hat{\sigma}^{MF,1} \right) \\
 & + \int_{\Phi_n} \left( d\hat{\sigma}^{VV} + d\hat{\sigma}^{MF,2} \right) + \int_{\Phi_{n+1}} d\hat{\sigma}^{VS} + \int_{\Phi_{n+2}} d\hat{\sigma}^S
 \end{aligned}$$

- ▶ Require three principal ingredients
  - ▶ Two-loop matrix elements  
explicit poles from loop integrals
  - ▶ One-loop matrix elements  
explicit poles from loop integral  
and implicit poles from real emission
  - ▶ Tree-level matrix elements  
implicit poles from real emissions
- ▶ Challenge: Construction of subtraction methods for RR and RV contribution



- ▶ Sector decomposition [Binoth,Heinrich 2004;Anastasiou,Melnikov,Petriello 2004]
  - ▶  $pp \rightarrow H, pp \rightarrow V$  [Anastasiou,Melnikov,Petriello; Bühler,Herzog,Lazopoulos,Müller]
- ▶ Antenna subtraction [Gehrmann,Gehrmann-DeRidder,Glover]
  - ▶  $e^+e^- \rightarrow 3\text{jets}$  [Gehrmann,Gehrmann-DeRidder,Glover,Heinrich,Weinzierl]
  - ▶  $pp \rightarrow 2\text{jets}$  [Gehrmann,Gehrmann-DeRidder,Glover,Pires]
- ▶  $q_T$  subtraction [Catani,Grazzini 2007]
  - ▶  $pp \rightarrow H, pp \rightarrow V, pp \rightarrow VH, pp \rightarrow \gamma\gamma$   
[Catani,Cieri,DeFlorian,Ferrera,Grazzini,Tramontano]
- ▶ Sector-improved subtraction [Czakon 2010;Boughezal,Melnikov,Petriello 2011]
  - ▶  $pp \rightarrow t\bar{t}$  [Czakon,Fiedler,Mitov]
  - ▶  $pp \rightarrow H+\text{jet}$  [Boughezal,Caola,Melnikov,Petriello,Schulze]
- ▶ Cutoff method based on  $N$ -jettiness [Boughezal,Focke,Liu,Petriello 2015]
  - ▶  $pp \rightarrow W+\text{jet}$  [Boughezal,Focke,Liu,Petriello]

[Bärnreuther, Czakon, Fiedler, Mitov 2013-2014]



- ▶ Fully differential calculation in sector-improved subtraction scheme
- ▶ Constrains gluon PDF at large  $x$  (unc. reduction 15-25%)

[Brucherseifer, Caola, Melnikov 2014]

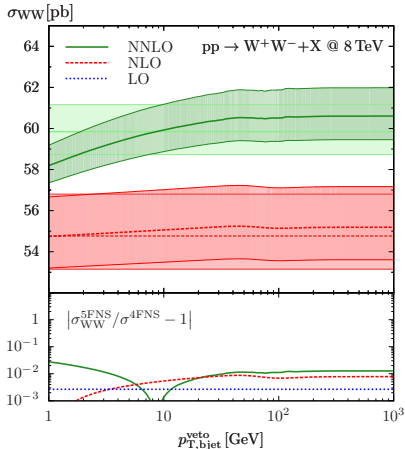
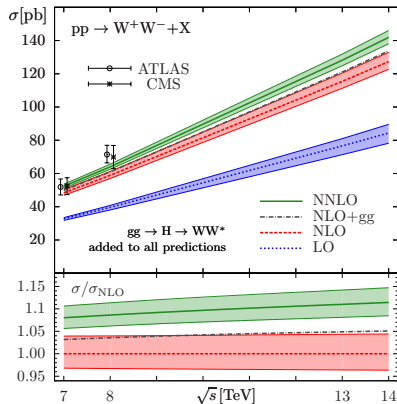
$p_{\perp}$	$\sigma_{\text{LO, pb}}$	$\sigma_{\text{NLO, pb}}$	$\delta_{\text{NLO}}$	$\sigma_{\text{NNLO, pb}}$	$\delta_{\text{NNLO}}$
0 GeV	$53.8^{+3.0}_{-4.3}$	$55.1^{+1.6}_{-0.9}$	+2.4%	$54.2^{+0.5}_{-0.2}$	-1.6%
20 GeV	$46.6^{+2.5}_{-3.7}$	$48.9^{+1.2}_{-0.5}$	+4.9%	$48.3^{+0.3}_{-0.02}$	-1.2%
40 GeV	$33.4^{+1.7}_{-2.5}$	$36.5^{+0.6}_{-0.03}$	+9.3%	$36.5^{+0.1}_{+0.1}$	-0.1%
60 GeV	$22.0^{+1.0}_{-1.5}$	$25.0^{+0.2}_{+0.3}$	+13.6%	$25.4^{+0.1}_{+0.2}$	+1.6%

$p_{\perp}$	$\sigma_{\text{LO, pb}}$	$\sigma_{\text{NLO, pb}}$	$\delta_{\text{NLO}}$	$\sigma_{\text{NNLO, pb}}$	$\delta_{\text{NNLO}}$
0 GeV	$29.1^{+1.7}_{-2.4}$	$30.1^{+0.9}_{-0.5}$	+3.4%	$29.7^{+0.3}_{-0.1}$	-1.3%
20 GeV	$24.8^{+1.4}_{-2.0}$	$26.3^{+0.7}_{-0.3}$	+6.0%	$26.2^{+0.01}_{-0.1}$	-0.4%
40 GeV	$17.1^{+0.9}_{-1.3}$	$19.1^{+0.3}_{+0.1}$	+11.7%	$19.3^{+0.2}_{+0.1}$	+1.0%
60 GeV	$10.8^{+0.5}_{-0.7}$	$12.7^{+0.03}_{+0.2}$	+17.6%	$12.9^{+0.1}_{+0.2}$	+1.6%

- ▶ Calculation performed in structure function approximation
- ▶ Fully differential using sector-improved subtraction
- ▶ Confirms NLO results at much higher theoretical accuracy

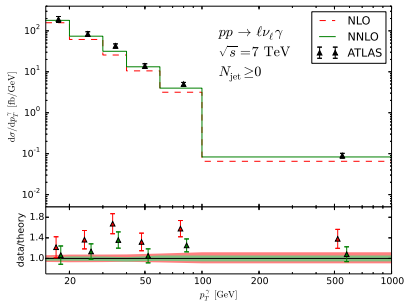
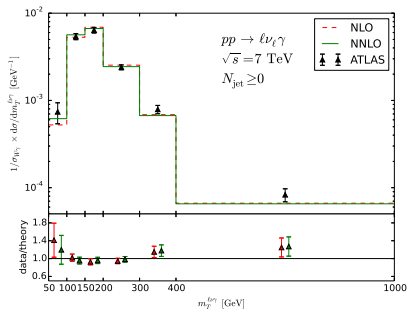


[Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Rathlev, Tancredi 2014]



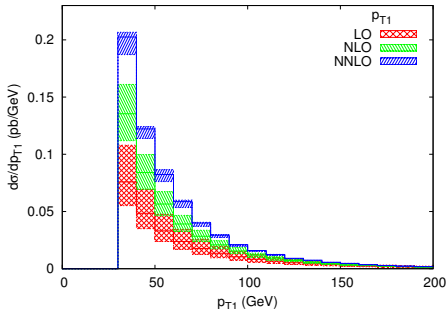
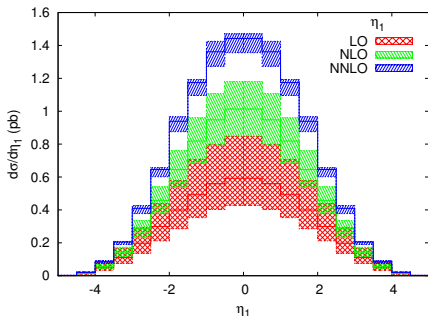
- ▶ Total cross section enhanced by 9(12)% at 7(14) TeV
- ▶ Top-subtracted 5FNS result agrees with 4FNS at 1(2)% for 7(14) TeV

[Grazzini, Kallweit, Rathlev 2015]



- ▶ Calculation performed using  $q_T$ -subtraction method
- ▶  $W\gamma \rightarrow$  NNLO effects important: 19% to 26%, depending on cuts
- ▶  $Z\gamma \rightarrow$  NNLO corrections 8% to 18%, depending on cuts

[Chen, Gehrmann, Glover, Jaquier 2014]

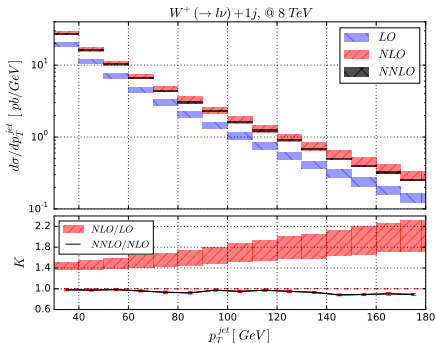
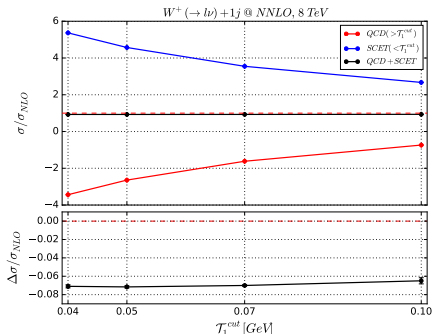


- ▶ Parton-level event generator, based on antenna subtraction
- ▶ Large rate change in inclusive result:

$$\sigma_{NLO} = 4.38_{-0.74}^{+0.76} pb \rightarrow \sigma_{NNLO} = 6.34_{-0.49}^{+0.28} pb \text{ at } p_{Tj} > 30 \text{ GeV}$$

- ▶ Residual theory uncertainty on  $p_T$ -spectra 5-16%
- ▶ Independent calculation by [Boughezal, Caola, Melnikov, Petriello, Schulze 2013]

[Boughezal, Focke, Liu, Petriello 2015]



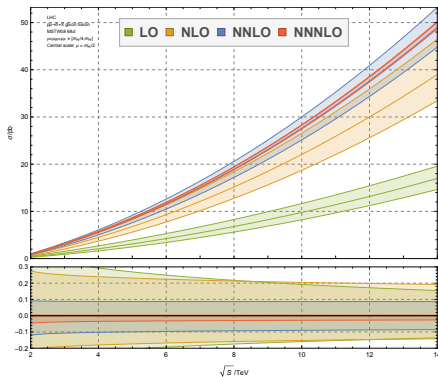
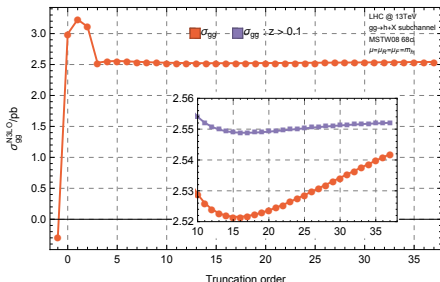
- ▶ New cutoff method based on  $N$ -jettiness (needed NNLL soft function)
- ▶ Techniques also applicable to Higgs-boson plus jet production

- ▶ Gluon fusion is dominant Higgs production mode at the LHC
- ▶ In large  $m_t$  limit described by effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}} - \frac{C}{4} H G_{\mu\nu}^a G_a^{\mu\nu}$$

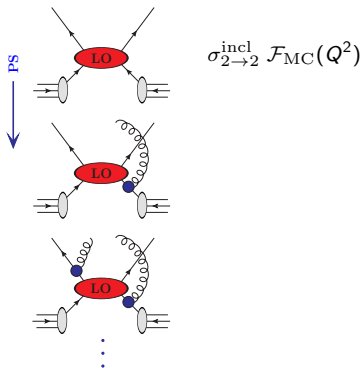
- ▶  $C$  known to N<sup>4</sup>LO [Chetyrkin,Kniehl,Steinhauser 1998], [Schröder,Steinhauser 2006], [Chetyrkin,Kühn,Sturm 2006]
- ▶ Inclusive and fully differential NNLO known [Anastasiou,Melnikov 2002], [Harlander,Kilgore 2002], [Anastasiou,Melnikov,Petriello 2005], [Catani,Grazzini 2007]
- ▶ Mixed QCD+EW corrections known [Anastasiou,Boughezal,Petriello 2009], [Actis,Passarino,Sturm,Uccirati 2008]
- ▶ NNLO scale uncertainty still  $\mathcal{O}(10\%)$   
Comparable to experimental uncertainty in Run I

[Anastasiou, Duhr, Dulat, Herzog, Mistlberger 2015]

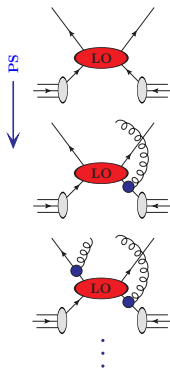


- ▶ First complete N<sup>3</sup>LO calculation at a hadron collider
- ▶ Total scale variation 3%, reducing theory uncertainty by factor 3
- ▶ Calculation performed using reverse unitarity and threshold expansion

► Parton showers



► Parton showers



$$\sigma_{2 \rightarrow 2}^{\text{incl}} \left[ \Delta(t_c, Q^2) \right.$$

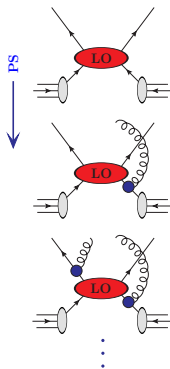
$$+ \int_{t_c}^{Q^2} \frac{dt}{t} \int dz \frac{\alpha_s}{2\pi} P(z) \Delta(t, Q^2)$$

$$+ \frac{1}{2} \left( \int_{t_c}^{Q^2} \frac{dt}{t} \int dz \frac{\alpha_s}{2\pi} P(z) \right)^2 \Delta(t, Q^2)$$

$$+ \dots$$



► Parton showers



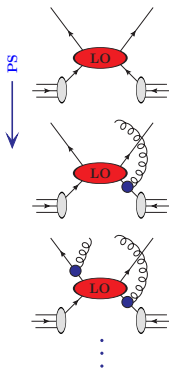
$$\sigma_{2 \rightarrow 2}^{\text{incl}} \left[ \Delta(\tau_c) \right.$$

$$+ \int_{\tau_c}^1 \frac{d\tau}{\tau} \int dz \frac{\alpha_s}{2\pi} P(z) \Delta(\tau)$$

$$+ \frac{1}{2} \left( \int_{\tau_c}^1 \frac{d\tau}{\tau} \int dz \frac{\alpha_s}{2\pi} P(z) \right)^2 \Delta(\tau)$$

$$+ \dots$$

► Parton showers



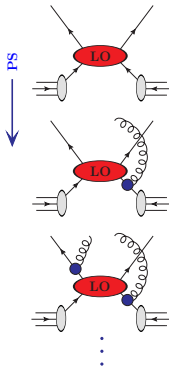
$$\sigma_{2 \rightarrow 2}^{\text{incl}} \left[ \Delta(\tau_c) \right.$$

$$+ \int_{\tau_c}^1 d\tau \frac{\alpha_s}{\tau} (A \log \tau + B) \Delta(\tau)$$

$$+ \frac{1}{2} \left( \int_{\tau_c}^1 d\tau \frac{\alpha_s}{\tau} (A \log \tau + B) \right)^2 \Delta(\tau)$$

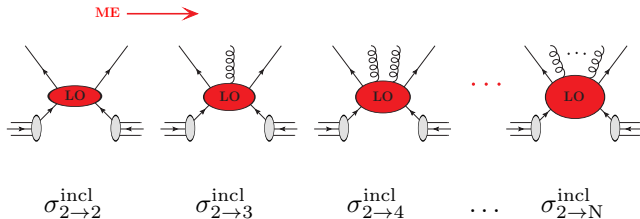
$$+ \dots$$

► Parton showers

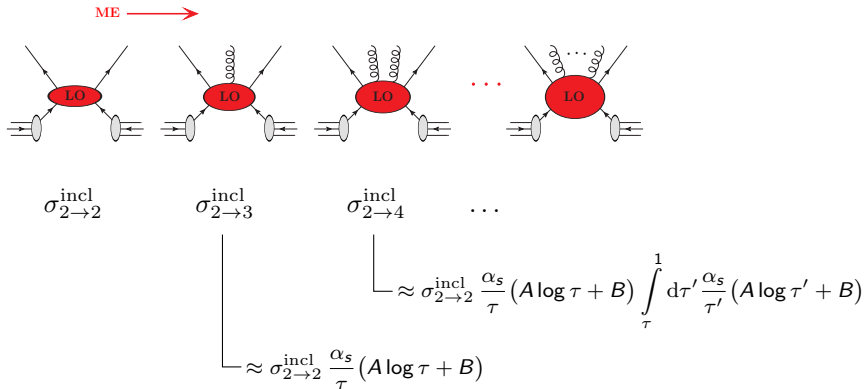


$$\begin{aligned}
 \sigma_{2 \rightarrow 2}^{\text{incl}} & \left[ \exp \left\{ - \int_{\tau_c}^1 d\tau' \frac{\alpha_s}{\tau'} (A \log \tau' + B) \right\} \right. \\
 & + \int_{\tau_c}^1 d\tau \frac{\alpha_s}{\tau} (A \log \tau + B) \exp \left\{ - \int_{\tau_c}^1 d\tau' \frac{\alpha_s}{\tau'} (A \log \tau' + B) \right\} \\
 & + \frac{1}{2} \left( \int_{\tau_c}^1 d\tau \frac{\alpha_s}{\tau} (A \log \tau + B) \right)^2 \exp \left\{ - \int_{\tau_c}^1 d\tau' \frac{\alpha_s}{\tau'} (A \log \tau' + B) \right\} \\
 & + \dots
 \end{aligned}$$

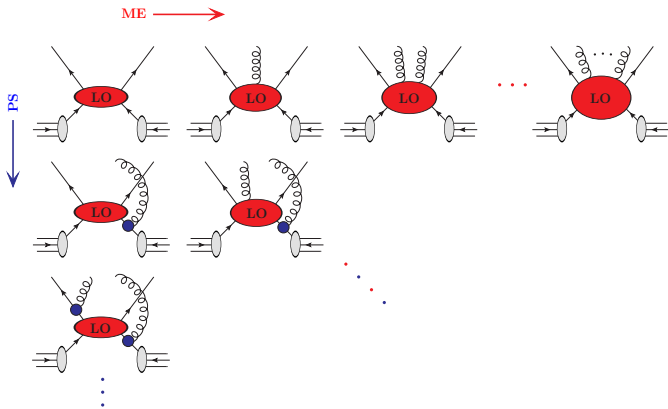
► Matrix elements



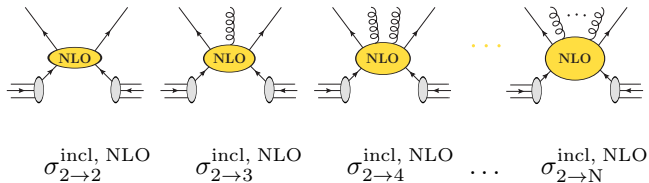
► Matrix elements



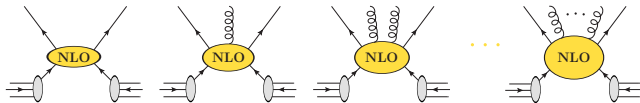
► Matrix element - parton shower merging (MEPS)



► NLO calculations



► NLO calculations



$$\sigma_{2 \rightarrow 2}^{\text{incl, NLO}}$$

$$\sigma_{2 \rightarrow 3}^{\text{incl, NLO}}$$

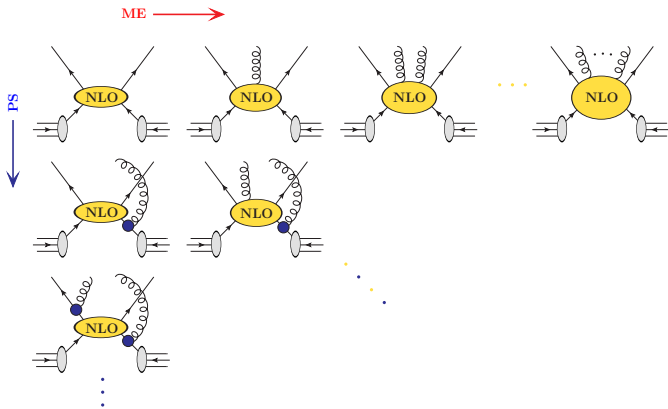
...

$$\left. \begin{array}{l} \sigma_{2 \rightarrow 2}^{\text{incl, NLO}} \\ \sigma_{2 \rightarrow 3}^{\text{incl, NLO}} \end{array} \right\} \approx \sigma_{2 \rightarrow 2}^{\text{incl}} \left( \frac{\alpha_s}{\tau} (A \log \tau + B) \left[ 1 + \frac{\alpha_s}{2\pi} C_1 + \frac{\alpha_s}{2\pi} \beta_0 \log \tau \right. \right. \\ \left. \left. - \int_{\tau}^1 d\tau' \frac{\alpha_s}{\tau'} (A \log \tau' + B) + \dots \right] + \frac{\alpha_s}{\tau} \frac{\alpha_s}{2\pi} K_g A \log \tau \right) + \dots$$

$$\left. \sigma_{2 \rightarrow 2}^{\text{incl}} \left[ 1 + \frac{\alpha_s}{2\pi} C_1 \right] \right\}$$

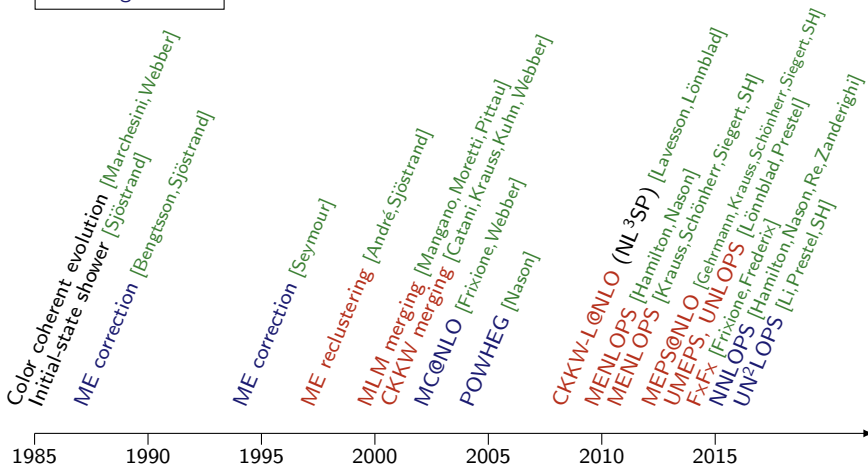


- ▶ Matrix element - parton shower merging at NLO (MEPS@NLO)

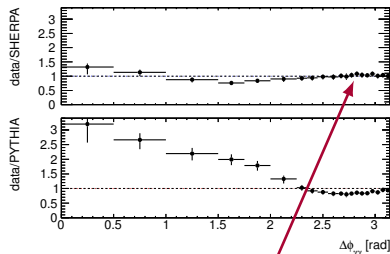
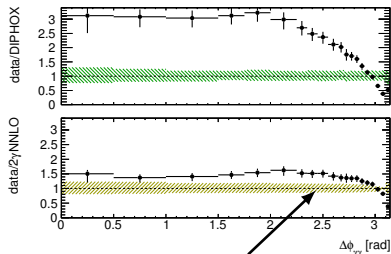
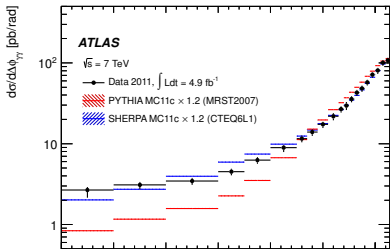
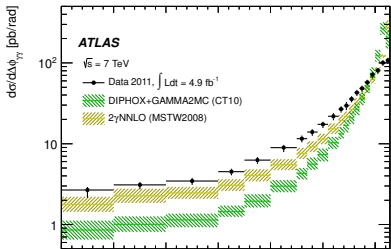


# Parton shower resummation and combination with $N^{\times}$ LO

Merging related  
Matching related



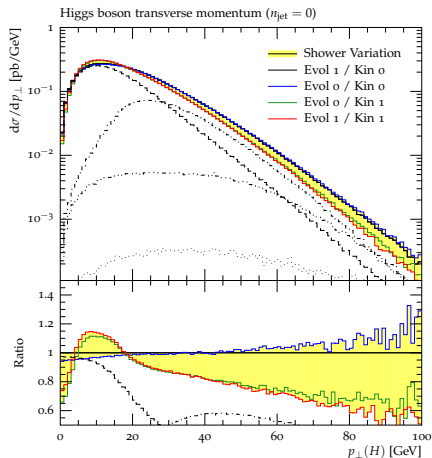
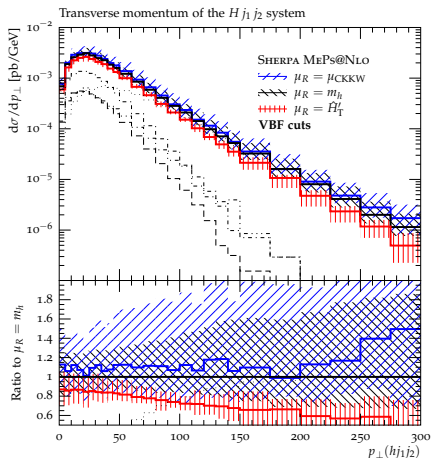
# Parton shower resummation and combination with N<sup>x</sup>LO



NNLO  $pp \rightarrow \gamma\gamma$

LO ME+PS merging

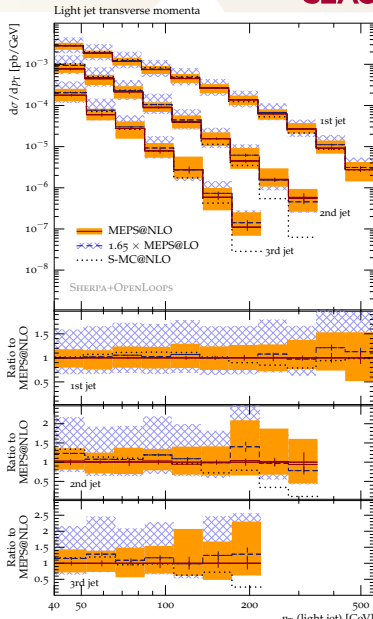
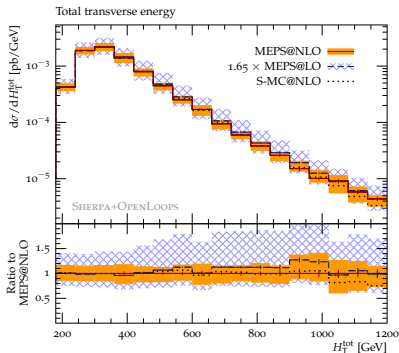
[Krauss, Schönerr, SH 2014]



- Combines NLO QCD calculations for  $pp \rightarrow h + 0, 1\&2\text{-jet}$  plus 3-jet at LO
- Resummation uncertainty remains large in vetoed region relevant for VBF

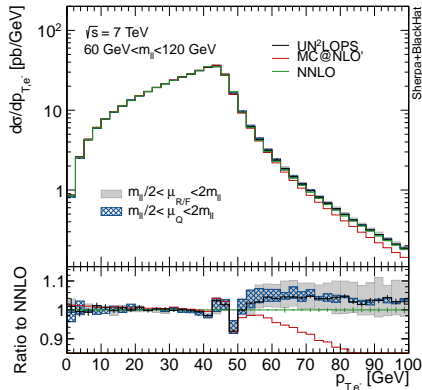
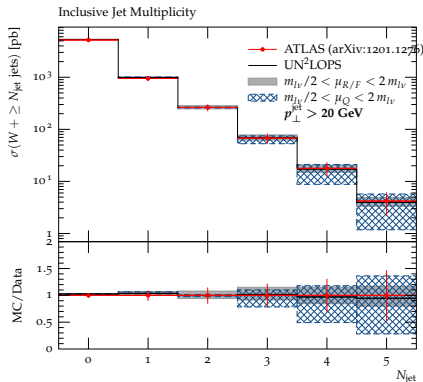
[Krauss, Maierhöfer, Pozzorini, Schönherr, Siegert, SH 2014]

- ▶ First matched/merged sim for  $t\bar{t}+2j$   
full result has  $t\bar{t}+0,1,2j@NLO$ ,  $3j@LO$
- ▶ Largely reduced theory uncertainty  
for both for measurement ( $p_T$ ,  $N_{jet}$ )  
and BSM search ( $H_T$ ) observables



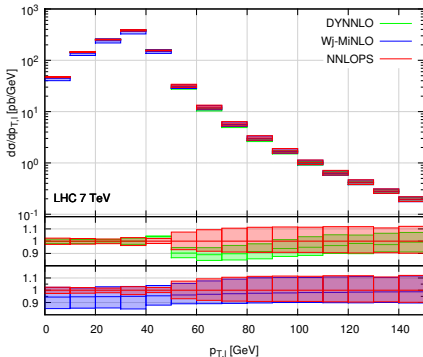
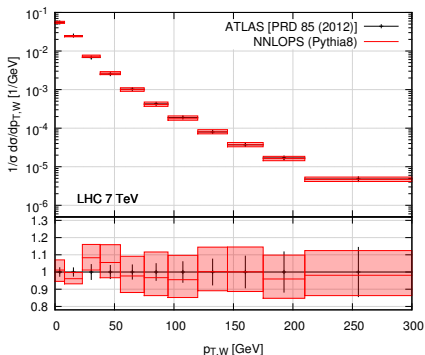
[Li, Prestel, SH 2014]

- ▶ Matching scheme based on unitarized merging method [Lönnblad, Prestel 2012]
- ▶ First NNLO+PS event generator for Drell-Yan type processes  
Includes dominant electroweak (QED) effects

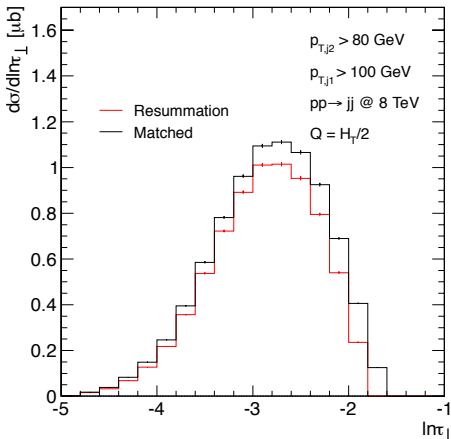
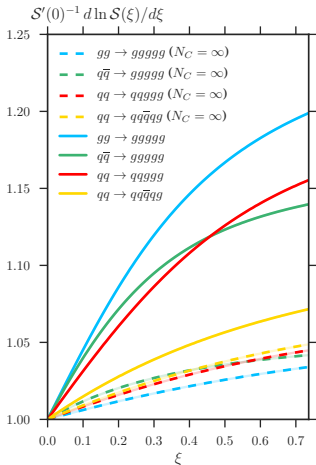


[Karlberg,Re,Zanderighi 2014]

- ▶ Matching scheme based on MiNLO method [Hamilton,Nason,Re,Zanderighi 2013]
- ▶ NNLO rate obtained by reweighting with fully differential  $K$ -factor



[Gerwick,SH,Marzani,Schumann 2014]



- ▶ Automated calculation of hard matrix and soft anomalous dimension
- ▶ Automated matching of spectrum at LO, based on dipole subtraction



- ▶ Precision QCD at hadron colliders is a reality
- ▶ NLO calculations, even at high multiplicity, are the standard
- ▶ Matching to parton showers extends NLO precision to the particle level
- ▶ NNLO calculations now become available for processes with light jets
- ▶ The first NNNLO result at a hadron collider was just computed
- ▶ Many higher-order results are implemented in event generators